CONICS

Circle: General form: $x^2 + y^2 + Dx + Ey + F = 0$

Center-radius form: $(x - h)^2 + (y - k)^2 = r^2$

Family: $x^2 + y^2 + D_1x + E_1y + F_1 + k(x^2 + y^2 + D_2x + E_2y + F_2) = 0$

If $k = 1$, the result will be a straight line called the *radical axis*. If the given circles intersect in two points, this line passes through them. If the circles intersect at one point, the radical axis is tangent at this point. If the circles have no common point, the radical axis is between them, perpendicular to a line joining their centers.

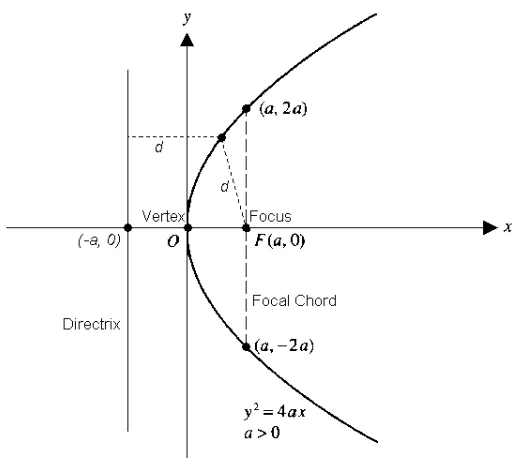
Parabola: Opening right if $a > 0$:

$y^2 = 4ax$	F(a, 0)	V(0, 0)
$(y - k)^2 = 4a(x - h)$	F($h + a$, k)	V(h, k)

Opening up if $a > 0$:

$x^2 = 4ay$	F(0, a)	V(0, 0)
$(x - h)^2 = 4a(y - k)$	F(h, $k + a$)	V(h, k)

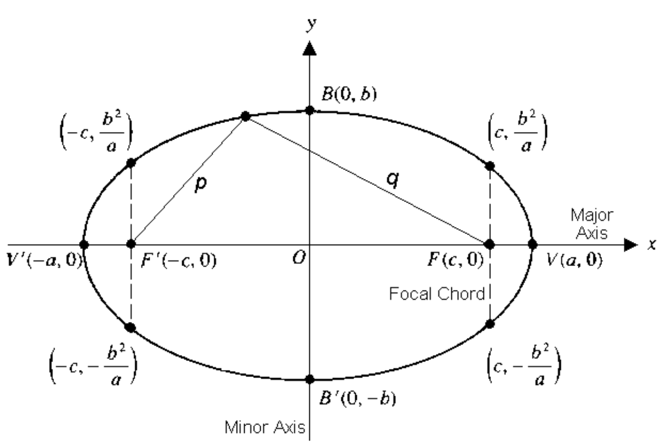
The eccentricity of a parabola = 1.



Elipse: *X*-axis orientation (reversed positions of *a* and *b* indicate *y*-axis orientation): $c^2 = a^2 - b^2$ $\quad p + q = 2a$

$a^2 > b^2 \qquad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$

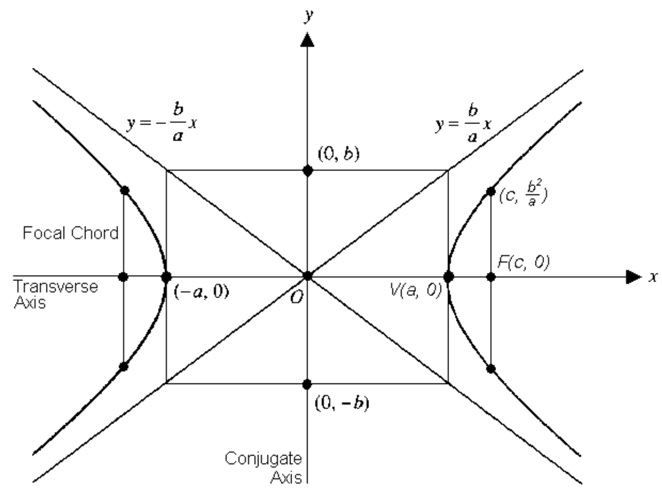
Eccentricity: $e = c/a$, $0 < e < 1$



Hyperbola: Aligned with *x*-axis (reverse *x* and *y* terms, or *x-h* and *y-k*, for *y*-axis alignment):

$c^2 = a^2 + b^2 \qquad \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$

Asymptotes: $\frac{x - h}{a} \pm \frac{y - k}{b} = 0$ Eccentricity: $e = \frac{c}{a}$, $e > 1$



Identifying Conics: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

If $B^2 - 4ac < 0$ then: ellipse, point or no graph

If $B^2 - 4ac > 0$ then: hyperbola or 2 intersecting lines

If $B^2 - 4ac = 0$ then: parabola, 2 parallel lines, 1 line, or no graph

Rotation and Translation of Conics:

If there is a *B* term, rotation is required.

If there is a *D* or *E* term, translation is required.

If $A = C$, $q = 45^\circ$ $\qquad$ If $A \neq C$, $\tan 2q = \frac{B}{A - C}$

$\cos 2q = 2\cos^2 q - 1$

$x = x'\cos q - y'\sin q$

$y = x'\sin q - y'\cos q$

Find *h*, *k* values for an equation with an *xy* term by substituting *x'* + *h* and *y'* + *k* for *x* and *y*. Perform multiplication. Factor out *x'* and *y'* and find values of *h* and *k* to eliminate them.

Find *h*, *k* values for an equation with squares by factoring the equation. The *h*, *k* values will appear reverse sign.

To translate an equation to a given origin, ADD the *h*, *k* values to the *x*, *y* values in the equation.

Distance of a Directrix of an elipse or hyperbola to its associated Focus: (used in Calculus III)

$$d = \frac{a - c}{e} + (a - c) \qquad \text{or} \qquad d = \frac{c(1 - e^2)}{e^2}$$

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