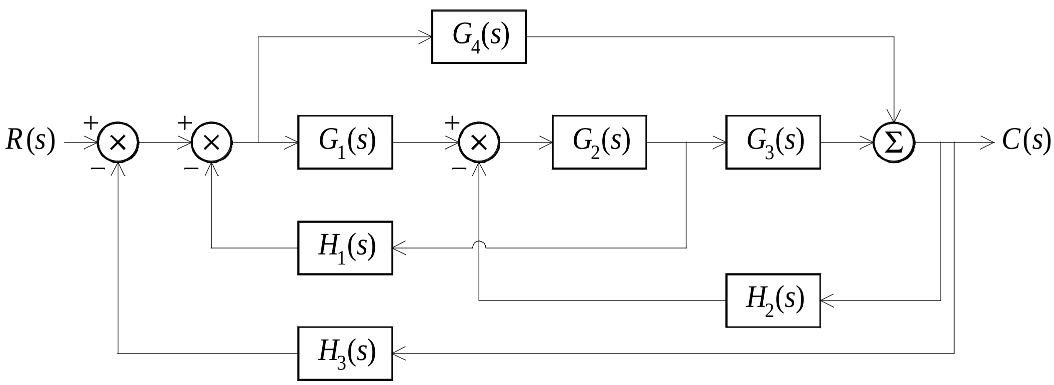
MASON'S GAIN RULE

An example of finding the transfer function of a system represented by a block diagram using Mason's rule. This problem has also been worked using a matrix solution; see the file MatrixSolution.pdf.

The Problem:

Given the system:



Mason's Gain Rule:

$$M = \frac{\sum_j M_j \Delta_j}{\Delta}$$

M = transfer function or gain of the system

M_j = gain of one forward path

j = an integer representing the forward paths in the system

Δ_j = 1 – the loops remaining after removing path j. If none remain, then Δ_j = 1.

Δ = 1 - Σ loop gains + Σ nontouching loop gains taken two at a time - Σ nontouching loop gains taken three at a time + Σ nontouching loop gains taken four at a time - · · ·

1) Find the forward paths and their gains:

A **forward path** is a path from $R(s)$ to $C(s)$ that does not cross the same point more than once. There are two forward paths in this example, so we have a j = 1 and a j = 2. The two paths are:

$$M_1 = G_1 G_2 G_3 \quad \text{and} \quad M_2 = G_4$$

2) Find the loops and their gains:

A loop is a closed path that can be negotiated without crossing the same point more than once. There are five loops in this example:

$\text{Loop } 1 = -G_1G_2H_1$
$\text{Loop } 2 = -G_2G_3H_2$
$\text{Loop } 3 = -G_1G_2G_3H_3$
$\text{Loop } 4 = -G_4H_3$
$\text{Loop } 5 = G_4H_2G_2H_1$

3) Find the Δ_js:

If we eliminate the path $M_1 = G_1G_2G_3$ from the system, no complete loops remain so: $\Delta_1 = 1$	If we eliminate the path $M_2 = G_4$ from the system, no complete loops remain so: $\Delta_2 = 1$

4) Find Δ:

Since there are no nontouching loop pairs in this example, Δ will just be one minus the sum of the loop gains:

$$\Delta = 1 - \left[\left(-G_1G_2H_1\right) + \left(-G_2G_3H_2\right) + \left(-G_1G_2G_3H_3\right) + \left(-G_4H_3\right) + \left(G_4H_2G_2H_1\right)\right]$$

The Solution:

$$\text{Mason's Rule: } M = \frac{\sum_j M_j \Delta_j}{\Delta}$$

Applying the formula for Mason's rule, we have the transfer function:

$$\frac{C(s)}{R(s)} = M = \frac{G_1G_2G_3 + G_4}{1 + G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3H_3 + G_4H_3 - G_4H_2G_2H_1}$$