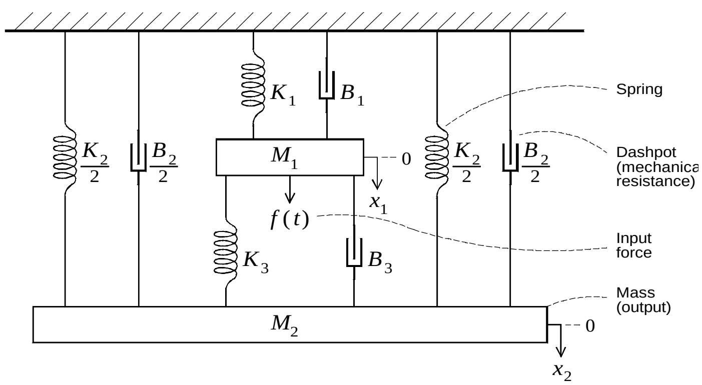
WRITING A TRANSFER FUNCTION FOR A MECHANICAL SYSTEM

The following example illustrates how to write a transfer function to describe a mechanical system.

THE PROBLEM

Given the mechanical system



find the transfer function $G(s)$ for the system $F(s) \longrightarrow \boxed{G(s)} \longrightarrow X_2(s)$

LIST THE FORCES ON EACH MASS

Forces on M_1:	$-K_1x_1$	$-B_1\dot{x}_1$
	$K_3(x_2 - x_1)$	$B_3(\dot{x}_2 - \dot{x}_1)$
	$f(t)$	
Forces on M_2:	$-K_2x_2$	$-B_2\dot{x}_2$
	$-K_3(x_2 - x_1)$	$-B_3(\dot{x}_2 - \dot{x}_1)$

WRITE THE DIFFERENTIAL EQUATIONS FOR EACH MASS

$$M_1\ddot{x}_1 = -K_1x_1 - B_1\dot{x}_1 + K_3(x_2 - x_1) + B_3(\dot{x}_2 - \dot{x}_1) + f(t)$$

$$M_2\ddot{x}_2 = -K_2x_2 - B_2\dot{x}_2 - K_3(x_2 - x_1) - B_3(\dot{x}_2 - \dot{x}_1)$$

CONVERT TO THE *S*-DOMAIN

$$M_1 s^2 X_1(s) = -K_1 X_1(s) + K_3\left[X_2(s) - X_1(s)\right] - B_1 s X_1(s) + B_3\left[sX_2(s) - sX_1(s)\right] + F(s)$$

$$M_2 s^2 X_2(s) = -K_2 X_2(s) - B_2 s X_2(s) - K_3\left[X_2(s) - X_1(s)\right] - B_3\left[sX_2(s) - sX_1(s)\right]$$

REARRANGE AS SUMS OF THE NET FORCES

$$M_1 s^2 X_1(s) + K_1 X_1(s) - K_3 X_2(s) - K_3 X_1(s) + B_1 s X_1(s) - B_3 s X_2(s) + B_3 s X_1(s) = F(s)$$

$$M_2 s^2 X_2(s) + K_2 X_2(s) + B_2 s X_2(s) + K_3 X_2(s) - K_3 X_1(s) + B_3 s X_2(s) - B_3 s X_1(s) = 0$$

PLACE IN MATRIX FORM

$$\begin{array}{ll} & \qquad\quad \text{X}_1 \text{ terms:} \qquad\qquad\qquad\qquad \text{X}_2 \text{ terms:} \\ \begin{array}{l}\text{M}_1 \text{ terms:} \\ \text{M}_2 \text{ terms:}\end{array} & \begin{bmatrix} M_1 s^2 + K_1 + B_1 s + K_3 + B_3 s & -K_3 - B_3 s \\ -K_3 - B_3 s & M_2 s^2 + K_2 + B_2 s + K_3 + B_3 s \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \begin{bmatrix} F(s) \\ 0 \end{bmatrix} \end{array}$$

SOLVE FOR *X*₂(*s*)

$$X_2(s) = \frac{\begin{vmatrix} M_1 s^2 + K_1 + B_1 s + K_3 + B_3 s & F(s) \\ -K_3 - B_3 s & 0 \end{vmatrix}}{\begin{vmatrix} M_1 s^2 + K_1 + B_1 s + K_3 + B_3 s & -K_3 - B_3 s \\ -K_3 - B_3 s & M_2 s^2 + K_2 + B_2 s + K_3 + B_3 s \end{vmatrix}}$$

$$X_2(s) = \frac{-F(s)\left[-K_3 - B_3 s\right]}{\left[M_1 s^2 + K_1 + B_1 s + K_3 + B_3 s\right]\left[M_2 s^2 + K_2 + B_2 s + K_3 + B_3 s\right] - \left(-K_3 - B_3 s\right)^2}$$

SOLVE FOR *G*(*s*)

$$G(s) = \frac{X_2(s)}{F(s)} = \frac{K_3 + B_3 s}{\left[M_1 s^2 + K_1 + B_1 s + K_3 + B_3 s\right]\left[M_2 s^2 + K_2 + B_2 s + K_3 + B_3 s\right] - \left(-K_3 - B_3 s\right)^2}$$

We have solved for the transfer function of the system, $G(s)$. If we instead solved for $X_2(s)$ and converted this to the time domain as $x_2(t)$, this would be the position of the mass M_s as a function of time t. If we multiplied the expression for $X_2(s)$ by s before converting to the time domain, this would be the equivalent of taking the derivative in the time domain and the result would be the velocity of M_2.