TRIGONOMETRIC FUNCTIONS

Chapter 8

Degrees	Radians	sin θ	cos θ	tan θ	cot θ	sec θ	csc *q*
0°	0	0	1	0	Undefined	1	Undefined
30°	$\pi/6$	1/2	$\sqrt{3}/2$	$\sqrt{3}/3$	$\sqrt{3}$	$2\sqrt{3}/3$	2
45°	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1	1	$\sqrt{2}$	$\sqrt{2}$
60°	$\pi/3$	$\sqrt{3}/2$	1/2	$\sqrt{3}$	$\sqrt{3}/3$	2	$2\sqrt{3}/3$
90°	$\pi/2$	1	0	Undefined	0	Undefined	1
120°	$2\pi/3$	$\sqrt{3}/2$	-1/2	$-\sqrt{3}$	$-\sqrt{3}/3$	-2	$2\sqrt{3}/3$
135°	$3\pi/4$	$\sqrt{2}/2$	$-\sqrt{2}/2$	-1	-1	$-\sqrt{2}$	$\sqrt{2}$
150°	$5\pi/6$	1/2	$-\sqrt{3}/2$	$-\sqrt{3}/3$	$-\sqrt{3}$	$-2\sqrt{3}/3$	2
180°	π	0	-1	0	Undefined	-1	Undefined
210°	$7\pi/6$	-1/2	$-\sqrt{3}/2$	$\sqrt{3}/3$	$\sqrt{3}$	$-2\sqrt{3}/3$	-2
225°	$5\pi/4$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	1	1	$-\sqrt{2}$	$-\sqrt{2}$
240°	$4\pi/3$	$-\sqrt{3}/2$	-1/2	$\sqrt{3}$	$\sqrt{3}/3$	-2	$-2\sqrt{3}/3$
270°	$3\pi/2$	-1	0	Undefined	0	Undefined	-1
300°	$5\pi/3$	$-\sqrt{3}/2$	1/2	$-\sqrt{3}$	$-\sqrt{3}/3$	2	$-2\sqrt{3}/3$
315°	$7\pi/4$	$-\sqrt{2}/2$	$\sqrt{2}/2$	-1	-1	$\sqrt{2}$	$-\sqrt{2}$
330°	$11\pi/6$	-1/2	$\sqrt{3}/2$	$-\sqrt{3}/3$	$-\sqrt{3}$	$2\sqrt{3}/3$	-2
360°	2π	0	1	0	Undefined	1	Undefined

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Radian measure: 8.1 p420

$$1^\circ = \frac{\pi}{180}\text{ radians}$$

$$1\text{ radian} = \frac{180^\circ}{\pi}$$

Reduction formulas:

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta \qquad \tan(-\theta) = -\tan\theta$$

$$\sin(\theta) = -\sin(\theta - \pi) \quad \cos(\theta) = -\cos(\theta - \pi) \quad \tan(\theta) = \tan(\theta - \pi)$$

The six trigonometric functions:

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{r} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{r}{y} = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{r}{x} = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{y}{x} = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{x}{y} = \frac{1}{\tan\theta}$$

Sum or difference of two angles:

$$\sin(a \pm b) = \sin a\cos b \pm \cos a\sin b$$

$$\cos(a \pm b) = \cos a\cos b \mp \sin a\sin b$$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a\tan b}$$

Double angle formulas:

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$\sin 2\theta = 2\sin\theta\cos\theta \qquad \cos 2\theta = 2\cos^2\theta - 1$$

$$\cos 2\theta = 1 - 2\sin^2\theta \qquad \cos 2\theta = \cos^2\theta - \sin^2\theta$$

Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta \qquad \cot^2\theta + 1 = \csc^2\theta$$

Half angle formulas:

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta) \qquad \cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$$

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{2}} \qquad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1 + \cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}} = \frac{\sin\theta}{1 + \cos\theta} = \frac{1 - \cos\theta}{\sin\theta}$$

Sum and product formulas:

$$\sin a\cos b = \tfrac{1}{2}[\sin(a + b) + \sin(a - b)]$$

$$\cos a\sin b = \tfrac{1}{2}[\sin(a + b) - \sin(a - b)]$$

$$\cos a\cos b = \tfrac{1}{2}[\cos(a + b) + \cos(a - b)]$$

$$\sin a\sin b = \tfrac{1}{2}[\cos(a - b) - \cos(a + b)]$$

$$\sin a + \sin b = 2\sin\left(\tfrac{a+b}{2}\right)\cos\left(\tfrac{a-b}{2}\right)$$

$$\sin a - \sin b = 2\cos\left(\tfrac{a+b}{2}\right)\sin\left(\tfrac{a-b}{2}\right)$$

$$\cos a + \cos b = 2\cos\left(\tfrac{a+b}{2}\right)\cos\left(\tfrac{a-b}{2}\right)$$

$$\cos a - \cos b = -2\sin\left(\tfrac{a+b}{2}\right)\sin\left(\tfrac{a-b}{2}\right)$$

Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos A$

where A is the angle of a scalene triangle opposite side a.

Limits of trigonometric functions: 8.2 p434

$$\lim_{x \to 0} \frac{\sin x}{x} = 1 \qquad \lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$

Integration Formulas p453,68,73 Differentiation Formulas p463,6

$\int \sin u \, du = -\cos u + C$	$\frac{d}{dx}(\sin u) = u' \cos u$
$\int \cos u \, du = \sin u + C$	$\frac{d}{dx}(\cos u) = -u' \sin u$
$\int \sec u \, du = \ln\lvert \sec u + \tan u \rvert + C$	
$\int \sec^2 u \, du = \tan u + C$	$\frac{d}{dx}(\tan u) = u' \sec^2 u$
$\int \sec u \tan u \, du = \sec u + C$	$\frac{d}{dx}(\sec u) = u' \sec u \tan u$
$\int \csc u \, du = -\ln\lvert \csc u + \cot u \rvert + C$	
$\int \csc^2 u \, du = -\cot u + C$	$\frac{d}{dx}(\cot u) = -u' \csc^2 u$
$\int \csc u \cot u \, du = -\csc u + C$	$\frac{d}{dx}(\csc u) = -u' \csc u \cot u$
$\int \tan u \, du = -\ln\lvert \cos u \rvert + C$	$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$
$\int \cot u \, du = \ln\lvert \sin u \rvert + C$	$\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$
$\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$	$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$
$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$	$\frac{d}{dx}[\text{arc} \cot u] = \frac{-u'}{1+u^2}$
$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \text{arc}\sec \frac{\lvert u \rvert}{a} + C$	$\frac{d}{dx}[\text{arc} \sec u] = \frac{u'}{\lvert u \rvert \sqrt{u^2 - 1}}$
$\int u^n u' = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$	$\frac{d}{dx}[\text{arc} \csc u] = \frac{-u'}{\lvert u \rvert \sqrt{u^2 - 1}}$

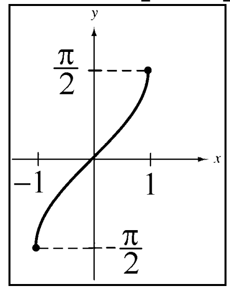
The **Power Rule** with Trigonometric Functions:

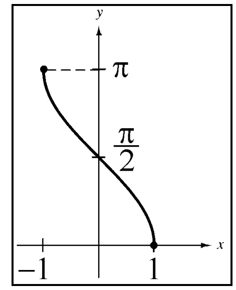
$$\frac{d}{dx}(\sin^3 x) = \frac{d}{dx}(\sin x)^3 = 3(\sin x)^2 \cos x = 3\sin^2 x \cos x$$

Definition of inverse trigonometric functions:

$y = \arcsin x$, iff $\sin y = x$ $\qquad$ $y = \arccos x$, iff $\cos y = x$

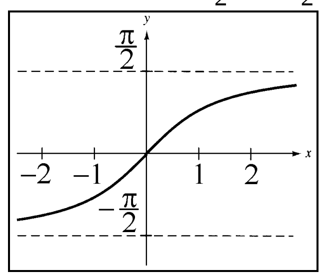
$-1 \le x \le 1 \quad -\frac{\boldsymbol{p}}{2} \le y \le \frac{\boldsymbol{p}}{2}$ $\qquad$ $-1 \le x \le 1 \quad 0 \le y \le \boldsymbol{p}$

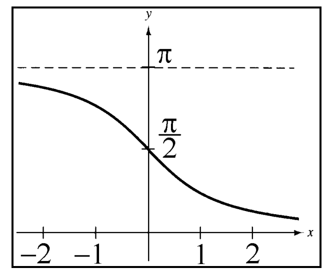




$y = \arctan x$, iff $\tan y = x$ $\qquad$ $y = \text{arccot}\, x$, iff $\cot y = x$

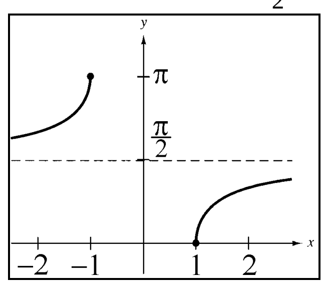
$-\infty < x < \infty \quad -\frac{\pi}{2} < y < \frac{\pi}{2}$ $\qquad$ $-\infty < x < \infty \quad 0 < y < \pi$

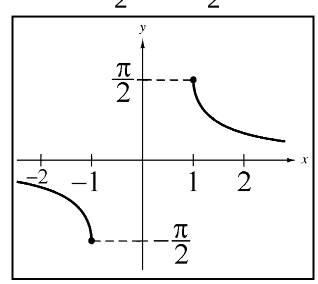




$y = \text{arcsec}\, x$, iff $\sec y = x$ $\qquad$ $y = \text{arccsc}\, x$, iff $\csc y = x$

$\lvert x \rvert \ge 1 \quad 0 \le y \le \pi,\ y \neq \frac{\pi}{2}$ $\qquad$ $\lvert x \rvert \ge 1 \quad -\frac{\pi}{2} \le y \le \frac{\pi}{2},\ y \neq 0$





Inverse Properties: 8.5 p461

If $-1 \le x \le 1$ and $-\pi/2 \le y \le \pi/2$, then

$\sin(\arcsin x) = x$ and $\arcsin(\sin y) = y$

If $-\pi/2 < y < \pi/2$, then

$\tan(\arctan x) = x$ and $\arctan(\tan y) = y$

If $\lvert x \rvert \le 1$ and $0 \le y < \pi/2$ or $\pi/2 < y \le \pi$, then

$\sec(\text{arcsec}\, x) = x$ and $\text{arcsec}(\sec y) = y$

Newton's Method is used to approximate the zeros of a function. Set up a table as follows: 8.3 p444

n	x_n	$f(x_n)$	$f'(x_n)$	$\frac{f(x_n)}{f'(x_n)}$	$x_n - \frac{f(x_n)}{f'(x_n)}$

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