SOLVING FIRST ORDER DIFFERENTIAL EQUATIONS

Methods for solving Linear, Exact, Separable, Homogeneous, and Bernoulli types

Linear

1. Rewrite in the form

   $$\boxed{\tfrac{dy}{dx} + (\text{terms of } x)y = (\text{terms of } x)}$$

2. Determine the integrating factor $e^{\int P(x)dx}$, where $P(x)$ is the factor multiplied by y above.
3. Multiply the equation by the integrating factor.
4. Discard the center term of the expression. (This happens by integrating and taking the derivative.)
5. Rewrite the left term of the equation in the form $\tfrac{d}{dx}(\text{new terms of } x)y$.
6. Integrate the equation with respect to x. Do this by removing the $\tfrac{d}{dx}$ from the left side and adding the $\int \; dx$ notation to the right. There will be a constant of integration in the result.
7. Solve for $y(x)$.

Exact

1. Rewrite in the form

   $$\boxed{(\text{terms of } x \text{ and } y)dx + (\text{terms of } x \text{ and } y)dy = 0}$$

   that is: $Mdx + Ndy = 0$.

2. The equation is ***exact*** if the partial derivative of M with respect to y equals the partial derivative of N with respect to x; $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.
3. Integrate M with respect to x and use $g(y)$ as the constant.
4. Differentiate the result with respect to y; the constant $g(y)$ becomes $g'(y)$.
5. Set this result equal to N to determine $g'(y)$.
6. $\int g'(y)\,dy$ to find the value of $g(y)$. This value does include a constant.
7. Substitute this value into the previous expression containing $g(y)$ and you have the solution $F(x,y)$.
8. The general solution is formed by setting the constant equal to the remaining expression.

Separable

1. Rewrite in the form

   $$\boxed{(\text{terms of } y)dy = (\text{terms of } x)dx}$$

2. Integrate both sides, consolidating the constants to a single constant on one side.
3. Solve for $y(x)$, the ***explicit*** solution. If this is not possible, then your result is an ***implicit*** solution.

Homogeneous

1. Rewrite in the form $\boxed{\tfrac{dy}{dx} = (\text{terms of } x \text{ and } y)}$
2. The presence of $\dfrac{y}{x}$ terms is required.
3. let $v = \dfrac{y}{x}$, then $y = vx$ and $\tfrac{dy}{dx} = v + x\tfrac{dv}{dx}$.
4. Manipulate until you get $(\text{terms of } v)dv = (\text{terms of } x)dx$.
5. Integrate both sides.

Bernoulli

1. Rewrite in the form

   $$\boxed{\tfrac{dy}{dx} + (\text{terms of } x)y = (\text{terms of } x)y^n}$$

2. This is a **Bernoulli** equation if n is not 0 or 1, otherwise it would be **linear**.
3. Let $v = y^{(1-n)}$. Find y. Find $\tfrac{dy}{dx}$ as $(\text{terms of } v)\tfrac{dv}{dx}$.
4. Perform the substitutions and rewrite the equation in the form $\tfrac{dv}{dx} + (\text{terms of } v) = (\text{terms of } x)$.
5. Hopefully we now have an equation in Linear form and can solve with that method.

Tom Penick tomzap@eden.com www.teicontrols.com/notes 12/5/97