

MATRIX DIFFERENTIAL EQUATIONS

Solving matrix first order differential equations given two vectors

25. p273

$$\mathbf{x}' = \begin{bmatrix} 4 & -3 \\ 6 & -7 \end{bmatrix} \mathbf{x}$$

$$\mathbf{x}_1 = \begin{bmatrix} 3e^{2t} \\ 2e^{2t} \end{bmatrix} \quad \mathbf{x}_2 = \begin{bmatrix} e^{-5t} \\ 3e^{-5t} \end{bmatrix}$$

First verify that the given vectors are solutions of the given system.

Then use the Wronskian to show that they are linearly independent.

Finally, write the general solution of the system.

We can verify that the given vectors are solutions of the given system by showing that the products of the coefficient matrix \mathbf{P} and the vector function are equal to the differentials of the vector functions.

$$\mathbf{P}\mathbf{x}_1 = \begin{bmatrix} 4 & -3 \\ 6 & -7 \end{bmatrix} \begin{bmatrix} 3e^{2t} \\ 2e^{2t} \end{bmatrix} = \begin{bmatrix} 6e^{2t} \\ 4e^{2t} \end{bmatrix} = \mathbf{x}_1' \quad \text{and} \quad \mathbf{P}\mathbf{x}_2 = \begin{bmatrix} 4 & -3 \\ 6 & -7 \end{bmatrix} \begin{bmatrix} e^{-5t} \\ 3e^{-5t} \end{bmatrix} = \begin{bmatrix} -5e^{-5t} \\ -15e^{-5t} \end{bmatrix} = \mathbf{x}_2'$$

We can show that the solutions \mathbf{x}_1 and \mathbf{x}_2 are linearly independent by showing that the Wronskian of the solutions is not equal to zero.

$$\begin{vmatrix} 3e^{2t} & e^{-5t} \\ 2e^{2t} & 3e^{-5t} \end{vmatrix} = 9e^{-3t} - 2e^{-3t} = 7e^{-3t} \neq 0$$

To find the general solution we use the formula	$\mathbf{x}(t) = c_1\mathbf{x}_1(t) + c_2\mathbf{x}_2(t)$
substituting \mathbf{x}_1 and \mathbf{x}_2	$\mathbf{x}(t) = c_1 \begin{bmatrix} 3e^{2t} \\ 2e^{2t} \end{bmatrix} + c_2 \begin{bmatrix} e^{-5t} \\ 3e^{-5t} \end{bmatrix}$
gives the general solution	$\mathbf{x}(t) = \begin{bmatrix} c_1 3e^{2t} & c_1 e^{-5t} \\ c_2 2e^{2t} & c_2 3e^{-5t} \end{bmatrix}$