**TWO-DIMENSIONAL SYSTEMS**

*Method for solving a two-dimensional first order system of differential equations*
from section 4.1 p228.

**13. p228**

\[ \begin{align*}
x' &= -2y \\
y' &= 2x
\end{align*} \quad \begin{align*}
x(0) &= 1 \\
y(0) &= 0
\end{align*} \]

Given a system of differential equations and initial conditions, find the general solutions and the particular solutions.

Take the derivative of the first expression

\[ x'' = -2y' \]

Substitute \(2x\) for \(y'\)

\[ x'' = -2(2x) \]

Now solve the resulting second order equation

\[ x'' + 4x = 0 \]

The characteristic equation is \( r^2 + 4 = 0 \) which has complex roots \( r = 0 \pm 2i \)

This gives us the general solution

\[ x(t) = e^{0t}(A \cos 2t + B \sin 2t) \]

Taking the derivative gives

\[ x'(t) = -2A \sin 2t + 2B \cos 2t \]

The first problem expression can be rewritten

\[ y = \frac{x'}{-2} \]

so that

\[ y(t) = -\frac{1}{2} x'(t) \]

by substituting for \(x'(t)\) we have

\[ y(t) = -\frac{1}{2}(-2A \sin 2t + 2B \cos 2t) \]

which reduces to

\[ y(t) = A \sin 2t + B \cos 2t \]

we now have this general solution to the system of equations to which we can apply the initial conditions

\[ \begin{align*}
x(t) &= (A \cos 2t + B \sin 2t) \\
y(t) &= A \sin 2t + B \cos 2t
\end{align*} \]

General solutions:

\[ x(t) = A \cos 2t + B \sin 2t \quad y(t) = A \sin 2t + B \cos 2t \]

Initial conditions:

\[ x(0) = 1 \quad y(0) = 0 \]

substituting:

\[ 1 = A \cos 0 + B \sin 0 \quad 0 = A \sin 0 + B \cos 0 \]

determines the constants:

\[ A = 1 \quad B = 0 \]

This yields the particular solutions:

\[ \begin{align*}
x(t) &= \cos 2t \\
y(t) &= \sin 2t
\end{align*} \]