1's Complement and 2's Complement Arithmetic

1's Complement Arithmetic

The Formula

\[ \overline{N} = (2^n - 1) - N \]

where:  
\( n \) is the number of bits per word  
\( N \) is a positive integer  
\( \overline{N} \) is -\( N \) in 1's complement notation  

For example with an 8-bit word and \( N = 6 \), we have:  
\[ \overline{N} = (2^8 - 1) - 6 = 255 - 6 = 249 = 11111001_2 \]

In Binary

An alternate way to find the 1's complement is to simply take the bit by bit complement of the binary number.  
For example:  
\[ N = +6 = 00000110_2 \]
\[ \overline{N} = -6 = 11111010_2 \]

Conversely, given the 1's complement we can find the magnitude of the number by taking its 1's complement.  
The largest number that can be represented in 8-bit 1's complement is 01111111_2 = 127 = $7F$.  The smallest is 10000000_2 = -127.  Note that the values 00000000_2 and 11111111_2 both represent zero.

Addition

End-around Carry.  When the addition of two values results in a carry, the carry bit is added to the sum in the rightmost position.  There is no overflow as long as the magnitude of the result is not greater than \( 2^n - 1 \).

2's Complement Arithmetic

The Formula

\[ N^* = 2^n - N \]

where:  
\( n \) is the number of bits per word  
\( N \) is a positive integer  
\( N^* \) is -\( N \) in 2's complement notation  

For example with an 8-bit word and \( N = 6 \), we have:  
\[ N^* = 2^8 - 6 = 256 - 6 = 250 = 11111010_2 \]

In Binary

An alternate way to find the 2's complement is to start at the right and complement each bit to the left of the first "1".  
For example:  
\[ N = +6 = 00000110_2 \]
\[ N^* = -6 = 11111010_2 \]

Conversely, given the 2's complement we can find the magnitude of the number by taking its 2's complement.  
The largest number that can be represented in 8-bit 2's complement is 01111111_2 = 127.  The smallest is 10000000_2 = -128.

Addition

When the addition of two values results in a carry, the carry bit is ignored.  There is no overflow as long as the is not greater than \( 2^n - 1 \) nor less than \( -2^n \).