OSCILLATION IN AN A-β BOX

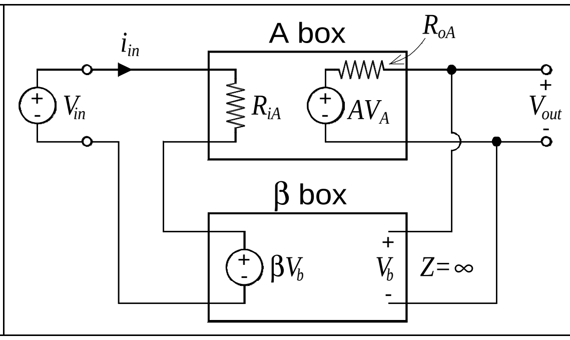
If a system contains a frequency of phase reversal (has 3 or more frequency corners) then oscillation may occur when there is a loop gain of at least 1, i.e. $\beta\left|A(j\omega)\right| = 1$ at the frequency of phase reversal.

The Problem:

The A-box is described by the following transfer function:

$$A(j\omega) = \frac{1000}{\left(1 + j\frac{f}{10^4}\right)\left(1 + j\frac{f}{10^5}\right)^2}$$

where A(jω) is the A box gain.



Find the frequency of phase reversal *f₀*

Use the expression:

$$-180 = -\tan^{-1}\frac{f_0}{10^4} - 2\tan^{-1}\frac{f_0}{10^5}$$

By trial and error, it can be determined that $\boldsymbol{f_0} = \mathbf{1.096 \times 10^5}$ **Hz**. This is the frequency of phase reversal.

Find the minimum value of β for which oscillation will occur

First, evaluate ***A*(jω)** at the frequency of phase reversal $\boldsymbol{f_0}$, and find the magnitude of the A-box gain.

$$A(j\omega) = \frac{1000}{\left(1 + j\frac{1.096\times10^5}{10^4}\right)\left(1 + j\frac{1.096\times10^5}{10^5}\right)^2}$$

$$\left|A(j\omega)\right| = 41.28$$

Now we find the value of β for which the magnitude of the loop gain is one.

$$1 = \beta\left|A(f_1)\right| = 41.28\beta$$

$$\boxed{\beta = 0.02422}$$

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What value of β would result in a gain margin of 10 dB?

In other words, what value of β would result in the loop gain being 10 dB **below** 0 dB at the frequency of phase reversal?

First, convert the value of **negative** 10 dB to units of V/V.

$$10^{(-10/20)} = 0.3162 \text{ V/V}$$

Then find the value of b that would result in this gain.

$$\beta|A(j\omega)| = 0.3162$$

$$\beta = \frac{0.3162}{0.4128}$$

$$\boxed{\beta = 0.7660}$$

Find the gain margin when β is given

If β is known to be 0.001, what is the gain margin? In other words, by how many dB is the loop gain **below** 0 dB at the frequency of phase reversal?

First, find the loop gain magnitude at phase reversal. Convert to decibels and change the sign to positive.

$$\beta|A(j\omega)| = 0.04128$$

$$20\log 0.04128 = -27.69 \text{ dB}$$

$$\boxed{\text{Gain Margin} = 27.69 \text{ dB}}$$

What value of β would result in a phase margin of 45°?

Use the expression:

$$-180 + 45 = -\tan^{-1}\frac{f_1}{10^4} - 2\tan^{-1}\frac{f_1}{10^5}$$

By trial and error, it can be determined that $\boldsymbol{f_1 = 5.285 \times 10^4}$ **Hz**. This is the frequency at which the amplifier is 45° from phase reversal.

Now we find the value of β for which the magnitude of the loop gain is one.

$$1 = \beta|A(f_1)| = \beta\left|\frac{1000}{\left(1 + j\frac{5.285\times10^4}{10^4}\right)\left(1 + j\frac{5.285\times10^4}{10^5}\right)^2}\right|$$

$$\boxed{\beta = 0.006881}$$