IMPULSE RESPONSE

Finding the Impulse Response of a linear time-invariant discrete-time (LTID) System

THE PROBLEM

Find the impulse response $h[k]$ of a system described by the equation:

$$\boxed{(E^2 - 6E + 9)\,y[k] = (E + 18)\,f[k]}$$

where $f[k]$ is the input and $y[k]$ is the output.

THE SOLUTION

From the equation given we can determine:

Characteristic polynomial:	$\lambda^2 - 6\lambda + 9 = (\lambda - 3)^2$
Characteristic roots:	$\lambda = 3,\ 3$
Characteristic modes:	$3^k,\ k3^k$
Zero-Input response:	$y_0[k] = c_1 3^k + c_2 k3^k$

Since $h[k]$ is the response to $\delta[k]$, the **impulse function**, substitute $h[k]$ for $y[k]$ and $\delta[k]$ for $f[k]$, and solve for $h[k]$.	$(E^2 - 6E + 9)h[k] = (E + 18)\delta[k]$ $h[k] = \dfrac{E + 18}{E^2 - 6E + 9}\delta[k]$	(A)

This is in the form $h[k] = \frac{b_0}{a_0}\delta[k]$ and for some reason we ignore the E values to get:	$b_0 = 18$ $a_0 = 9$

$u[k]$ is a unit step function which equals 1 for $k \geq 0$, and equals 0 for $k < 0$. The initial condition is multiplied by $u[k]$ because $h[k]$ is **causal**. So for the form $h[k] = \frac{b_0}{a_0}\delta[k] + y_0[k]u[k]$ we have:	$h[k] = 2\delta[k] + (c_1 3^k + c_2 k3^k)u[k]$	(B)

To find c_1 and c_2 we need 2 values of $h[k]$. Returning to equation (A):	$(E^2 - 6E + 9)h[k] = (E + 18)\delta[k]$	(A)
Since E^2 represents $k + 2$ and E represents $k + 1$, we rewrite:	$h[k+2] - 6h[k+1] + 9h[k] = \delta[k+1] + 18\delta[k]$	(C)

For k = -2 we have:	$h[0] - 6h[-1] + 9h[-2] = \delta[-1] + 18\delta[-2]$

We know that $h[-1] = h[-2] = 0$ and that $\delta[k] = 0$ except when $k = 0$, $\delta[0] = 1$, so:	$h[0]-6(0)+9(0)=0+18(0)$ $h[0]=0$

For $k = -1$, using equation (C): we have:	$h[k+2]-6h[k+1]+9h[k]=\delta[k+1]+18\delta[k]$ $h[1]-6h[0]+9h[-1]=\delta[0]+18\delta[-1]$ $h[1]-6(0)+9(0)=1+18(0)$ $h[1]=1$	(C)

Using equation (B): For $k = 0$ we have: For $k = 1$ we have:	$h[k]=2\delta[k]+(c_1 3^k + c_2 k 3^k)u[k]$ $0=2+[c_1+c_2(0)]1$ $0=2+c_1$ $c_1=-2$ $1=2(0)+[3c_1+3c_2]1$ $1=3(-2)+3c_2$ $7=3c_2$ $c_2=\frac{7}{3}$	(B)

THE ANSWER

Substituting into equation (B):	$h[k]=2\delta[k]+\left[(-2)3^k+\frac{7}{3}k(3^k)\right]u[k]$ $\boxed{h[k]=2\delta[k]-\left(2-\frac{7}{3}k\right)3^k u[k]}$