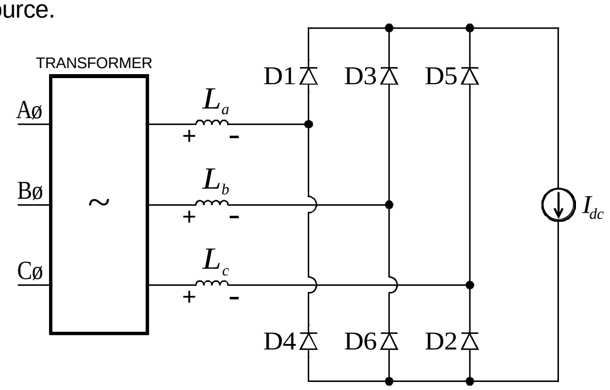
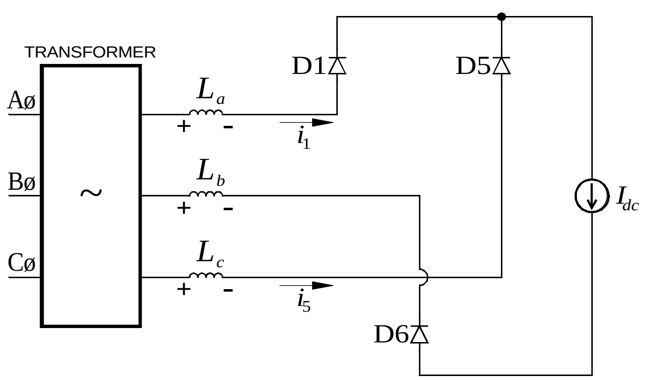
DERIVING KIMBARK'S EQUATIONS (with a)

3-PHASE RECTIFIER

Consider the 3-phase rectifier driving a constant current source.



Consider the commutation interval between D5 and D1, when D1 has come on and D5 has not yet gone off. At this time D1, D5, and D6 are on and the other diodes are off. So the circuit looks like this:



DERIVING THE 1ST EQUATION

Constant current output: $I_{dc} = i_1 + i_5$

Take the derivative: $\frac{dI_{dc}}{dt} = \frac{di_1}{dt} + \frac{di_5}{dt} = 0$

Therefore: $\frac{di_1}{dt} = -\frac{di_5}{dt}$

Kirchoff's voltage law:

$$-v_{ac} + L\frac{di_1}{dt} - L\frac{di_5}{dt} = 0$$

$$-v_{ac} + L\frac{di_1}{dt} + L\frac{di_1}{dt} = 0$$

$$-v_{ac} + 2L\frac{di_1}{dt} = 0$$

$$\boxed{\frac{di_1}{dt} = \frac{v_{ac}}{2L}}$$

v_{an}, v_{bn}, and v_{cn} are the transformer output voltages with respect to ground. Note that their sum is equal to zero.

Output voltage: $V_{DC} = v_{an} - L\frac{di_1}{dt} + L\frac{di_5}{dt} - v_{bn}$

$$V_{DC} = v_{an} - L\frac{v_{ac}}{2L} + 0 - v_{bn}$$

$$V_{DC} = v_{an} - \frac{1}{2}v_{ac} - v_{bn}$$

$$V_{DC} = v_{an} - \frac{1}{2}(v_{an} - v_{cn}) - v_{bn}$$

$$V_{DC} = \frac{1}{2}v_{an} + \frac{1}{2}v_{cn} - v_{bn}$$

Add zero: $+\left(-\frac{1}{2}v_{an} - \frac{1}{2}v_{cn} - \frac{1}{2}v_{bn}\right)$

$\boxed{V_{DC} = -\frac{3}{2}v_{bn}}$ (time dependent)

Explain about the 30°:

Line-to-line voltage: $v_{ac} = V_{LLp}\sin(\omega t)$

$$\frac{di_1}{dt} = \frac{v_{ac}}{2L} = \frac{V_{LLp}\sin(\omega t)}{2L}$$

Integrated: $i_1(t) = \frac{-V_{LLp}\cos(\omega t)}{2\omega L} + \text{const.}$

Boundary condition at the beginning of the commutation interval when D1 begins to conduct:

$$i_1(\omega t = \alpha) = 0$$

$$\frac{-V_{LLp}\cos\alpha}{2\omega L} + \text{const.} = 0$$

$$\text{const.} = \frac{V_{LLp}}{2\omega L}\cos\alpha$$

$$i_1(t) = \frac{-V_{LLp}\cos(\omega t)}{2\omega L} + \frac{V_{LLp}}{2\omega L}\cos\alpha$$

$$\boxed{i_1(t) = \frac{V_{LLp}}{2\omega L}\left[\cos\alpha - \cos(\omega t)\right]},\quad \alpha \le \omega t \le (\alpha + u)$$

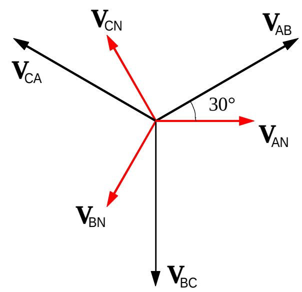
Boundary condition at the end of the commutation interval:

$$i_1(\omega t = \alpha + u) = I_{DC}$$

1st Kimbark Eq.: $\boxed{I_{DC} = \frac{V_{LLp}}{2\omega L}\left[\cos\alpha - \cos(\alpha + u)\right]}$

DERIVING THE 2ND EQUATION

The relation between line-to-line and line-to-neutral peak amplitudes:



$$V_{LLp} = \sqrt{3}V_{LNp}$$

$$\frac{3}{2}V_{LLp} = \frac{3}{2}\sqrt{3}V_{LNp}$$

$$\frac{\sqrt{3}}{2}V_{LLp} = \frac{3}{2}V_{LNp}$$

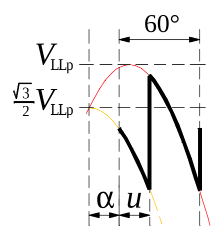
We want an expression for the average DC voltage V_{dc} and we already have the time-dependent expression:

$$V_{DC} = -\frac{3}{2}v_{bn}$$

The formula for average DC voltage is:

$$v_{\text{avg}} = \frac{1}{T}\int_{t_0}^{t_0+T} v(t)\,dt$$

The DC voltage waveform is periodic over 60° or π/3 radians. It is discontinuous at the beginning and end of the commutation interval. The DC voltage waveform is sinusoidal and the intervals of integration have been chosen so that the cosine function may be used.



$$V_{dc} = \frac{1}{\pi/3}\left[\int_{\alpha}^{\alpha+u}\frac{\sqrt{3}}{2}V_{LLp}\cos\theta\,d\theta + \int_{\alpha+u-\frac{\pi}{6}}^{\alpha+\frac{\pi}{6}} V_{LLp}\cos\theta\,d\theta\right]$$

$$= \frac{3V_{LLp}}{\pi}\left[\begin{matrix}\frac{\sqrt{3}}{2}\sin(\alpha+u) - \frac{\sqrt{3}}{2}\sin\alpha \\ +\sin\left(\alpha+\frac{\pi}{6}\right) - \sin\left(\alpha+\frac{-\pi}{6}+u\right)\end{matrix}\right]$$

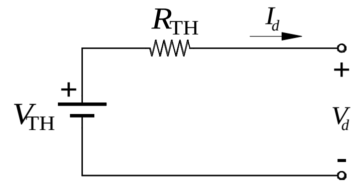
Identity: $\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$

$$= \frac{3V_{LLp}}{\pi}\left[\begin{matrix}\frac{\sqrt{3}}{2}\sin(\alpha+u) - \frac{\sqrt{3}}{2}\sin\alpha \\ +\sin\alpha\cos\frac{\pi}{6} + \cos\alpha\sin\frac{\pi}{6} \\ -\sin(\alpha+u)\cos\frac{\pi}{6} + \cos(\alpha+u)\sin\frac{\pi}{6}\end{matrix}\right]$$

2nd Kimbark Eq.: $$\boxed{V_{dc} = \frac{3V_{LLp}}{2\pi}\left[\cos\alpha + \cos(\alpha+u)\right]}$$

THE THÈVENIN EQUIVALENT

V_{TH} is the open circuit voltage. When $I_{dc} = 0$ then $u = 0$. Although we are taking I_{dc} to be constant, I suppose if the commutation interval was zero that would mean there could be no current flow at commutation because there is no initial current flow in the supply inductor connected to a newly opened diode.



Thèvenin voltage: $$V_{TH} = V_{dc}(u=0)$$

$$V_{TH} = \frac{3V_{LLp}}{2\pi}(\cos\alpha + \cos\alpha)$$

$$\boxed{V_{TH} = \frac{3V_{LLp}}{\pi}\cos\alpha}$$

Average voltage: $$V_{dc} = V_{TH} - I_{DC}R_{TH}$$

$$\frac{3V_{LLp}}{2\pi}\left[\cos\alpha + \cos(\alpha+u)\right] = \frac{3V_{LLp}}{\pi}\cos\alpha - \frac{V_{LLp}}{2\omega L}\left[\cos\alpha - \cos(\alpha+u)\right]R_{TH}$$

$$\frac{3}{\pi}\left[\cos\alpha + \cos(\alpha+u)\right] = \frac{6}{\pi}\cos\alpha - \frac{1}{\omega L}\left[\cos\alpha - \cos(\alpha+u)\right]R_{TH}$$

$$3\left[\cos\alpha + \cos(\alpha+u)\right] = 6\cos\alpha - \frac{\pi}{\omega L}\left[\cos\alpha - \cos(\alpha+u)\right]R_{TH}$$

$$\frac{\pi}{\omega L}\left[\cos\alpha - \cos(\alpha+u)\right]R_{TH} = 6\cos\alpha - 3\left[\cos\alpha + \cos(\alpha+u)\right]$$

$$\left[\cos\alpha - \cos(\alpha+u)\right]R_{TH} = \frac{3\omega L}{\pi}\left[\begin{matrix}2\cos\alpha - \cos\alpha \\ -\cos(\alpha+u)\end{matrix}\right]$$

$$\left[\cos\alpha - \cos(\alpha+u)\right]R_{TH} = \frac{3\omega L}{\pi}\left[\cos\alpha - \cos(\alpha+u)\right]$$

$$\boxed{R_{TH} = \frac{3\omega L}{\pi}}$$

DERIVING THE 3RD EQUATION

Power: $$P_{dc} = V_{dc}I_{dc}$$

$$P_{dc} = \frac{3V_{LLp}}{2\pi}\left[\cos\alpha + \cos(\alpha+u)\right]\frac{V_{LLp}}{2\omega L}\left[\cos\alpha - \cos(\alpha+u)\right]$$

3rd Kimbark Eq.: $$\boxed{P_{dc} = \frac{3V_{LLp}{}^2}{4\pi\omega L}\left[\cos^2\alpha - \cos^2(\alpha+u)\right]}$$