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EE368 Electrical Power Transmission and Distribution
Homework 2
2/12/99

Problem:

Consider a coaxial conductor with an outer to inner radius ratio of 2.7 and a relative permittivity of 2.5. R = 0, G = 0 (lossless line). $V_R = 100\ kV_{RMS} \angle 0°$, S = 10 MVA @ pf = 0.90 lagging, f = 60 Hz.

Plot the voltage magnitude and current magnitude over a distance of 5000 km.

Matlab Program:

```
% *************  GIVEN VALUES  ***********************************

format short                % 5-digit results
PwrOut = 10000000;          % Output Power
pf = .9;                    % Power Factor (lagging)
OutputVoltage = 100000;     % (phase angle 0)
Frequency = 60;             % Frequency 60 [Hz]
HeightAboveGround = 10;     % Height of transmission line [m]
RadiusRatio = 2.7;          % Outer radius/Inner radius
Length = 5000000;           % length of line [m] (5000 kilometers)
Epsilon = 8.85e-12;         % Permittivity of free space constant [F/m]
RelativePermittivity = 2.5  % Relative permittivity
Mu = 4*pi*10^-7;            % constant [T·m/A]
Omega = Frequency*2*pi      % Frequency converted to radians
R = 0;                      % Resistance per meter [Ohms/m]
G = 0;                      % Conductance [V/m]


% **************  CALCULATIONS  *********************************

%Output Power – Use to obtain Output Current
RadianPwrOut = -acos(pf)          %Phase angle
DegreesPowerOutput = rad2deg(RadianPwrOut)
Real=PwrOut*cos(RadianPwrOut);
Cplx=PwrOut*sin(RadianPwrOut);
% Convert two real variables to a complex number
VectorPwrOut = j*Cplx+Real;
OutputCurrent = VectorPwrOut/OutputVoltage

% Capacitance per meter of length
C = (Epsilon*RelativePermittivity*2*pi)/log(RadiusRatio)

% Inductance per meter of length
L = (Mu/(2*pi))*log(RadiusRatio)
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% Surge impedance
Zo = ((j*Omega*L)/(j*Omega*C))^(1/2)

% Attenuation constant, Alpha
Alpha = (R/2)*(C/L)^(1/2) + (G/2)*(L/C)^(1/2)

% Phase constant, Beta
Beta = Omega * (L*C)^(1/2)

% Propagation constant, Gamma
Gama = Alpha + j*Beta

% Wavelength, Lambda [m]
Lambda = 2*pi/(Omega*(L*C)^(1/2))


% ************  CREATE PLOTS FOR VOLTAGE MAGNITUDE,  ***************
%               CURRENT MAGNITUDE, AND PHASE

% CP2MP() returns the magnitude of a complex number
% phase() returns the phase of a complex number in radians
% LineVoltage(x,Gama,Zo,Vout,Iout)
%   Returns the voltage as a function of distance, x, along
%   a transmission line with characteristics Gama (gamma)
%   and Zo, having an output of Vout and Iout.
% LineCurrent(x,Gama,Zo,Vout,Iout)
%   Returns the curent as a function of distance, x, along
%   a transmission line with characteristics Gama (gamma)
%   and Zo, having an output of Vout and Iout.

newplot
x = -5000000 : 1000 : 0; axis([-5000 0 -180 100])
V = LineVoltage(x,Gama,Zo,OutputVoltage,OutputCurrent);
plot(x/1000,CP2MP(V)/1000,'k-'); grid on      % Plot voltage
xlabel('Distance in Kilometers')        % label the x-axis
ylabel('Kilovolts')                     % label the y-axis

figure; newplot;                      %opens new window for second plot
x = -5000000 : 1000 : 0; axis([-5000 0 -180 250])
I = LineCurrent(x,Gama,Zo,OutputVoltage,OutputCurrent);
plot(x/1000,CP2MP(I),'k-'); grid on      % Plot current
xlabel('Distance in Kilometers')        % label the x-axis
ylabel('Current in Amps')           % label the y-axis

figure; newplot;                    % While we're at it, plot phase
hold on; x = -5000000 : 80 : 0; axis([-5000 0 0 650])
V = LineVoltage(x,Gama,Zo,OutputVoltage,OutputCurrent);
I = LineCurrent(x,Gama,Zo,OutputVoltage,OutputCurrent);
plot(x/1000,720+phase(I)*180/pi,'k-',x/1000,720+phase(V)*180/pi,'k:');
x = -5000000 : 200000 : 0;
V = LineVoltage(x,Gama,Zo,OutputVoltage,OutputCurrent);
I = LineCurrent(x,Gama,Zo,OutputVoltage,OutputCurrent);
plot(x/1000,720+phase(I)*180/pi,'k+',x/1000,720+phase(V)*180/pi,'ko');
hold off; grid on
xlabel('Distance in Kilometers      +-+-Current     o-o-Volts')  % x-axis
ylabel('Phase in Degrees')          % label the y-axis
```

```
% **********  CREATE MATRIX OF VALUES FOR DATA SHEET  **************

x = 0;                                % Counter (meters)
Kilometers = []; VoltageMagnitude = []; VoltagePhaseAngle = [];
CurrentMagnitude = []; CurrentPhaseAngle = [];
while x > -Length-1
Vcomplex = LineVoltage(x,Gama,Zo,OutputVoltage,OutputCurrent);
Icomplex = LineCurrent(x,Gama,Zo,OutputVoltage,OutputCurrent);
Kilometers = [Kilometers;x/1000];
VoltageMagnitude = [VoltageMagnitude;CP2MP(Vcomplex)/1000];   % [KV]
VoltagePhaseAngle = [VoltagePhaseAngle;phase(Vcomplex)*180/pi];
CurrentMagnitude = [CurrentMagnitude;CP2MP(Icomplex)];        % [A]
CurrentPhaseAngle = [CurrentPhaseAngle;phase(Icomplex)*180/pi];
x = x – 100000;                       % Decrement the counter
end
format bank                           % Results to 2 decimal places
Output = [Kilometers VoltageMagnitude VoltagePhaseAngle CurrentMagnitude
CurrentPhaseAngle]
```

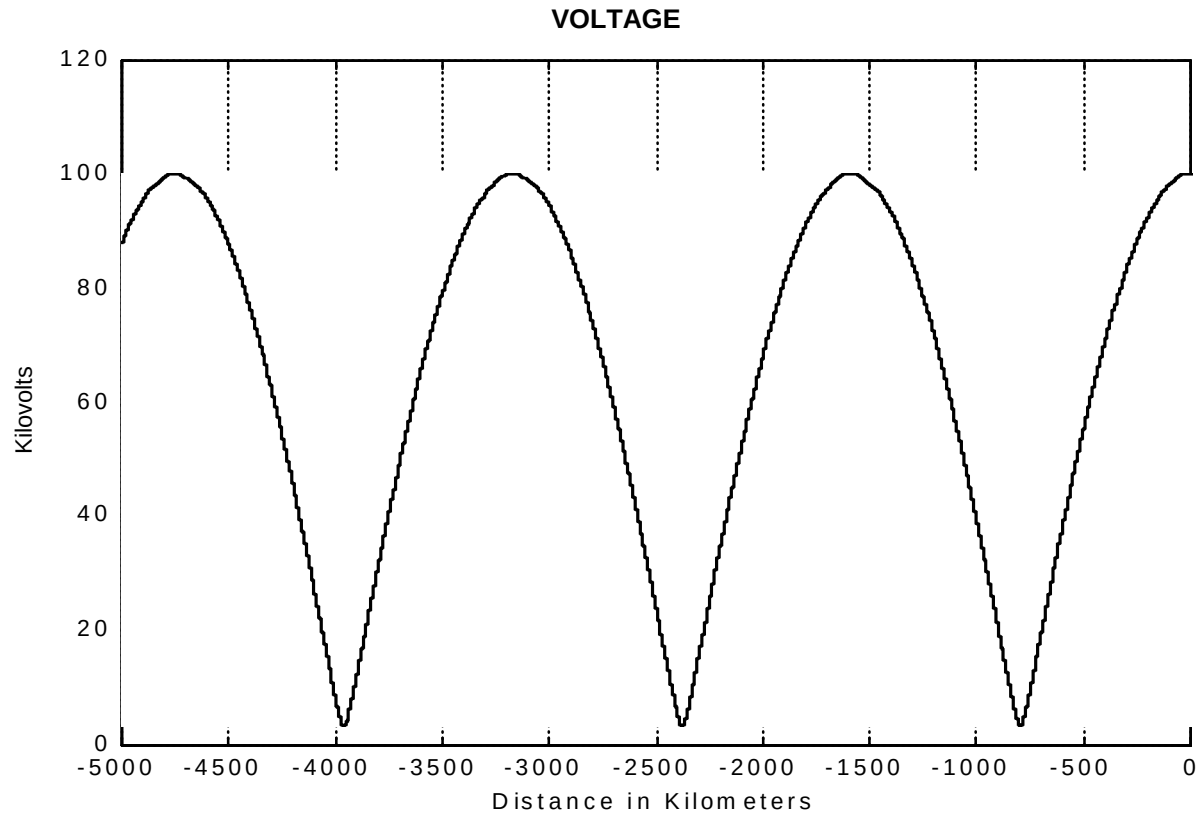
Formulas Used:

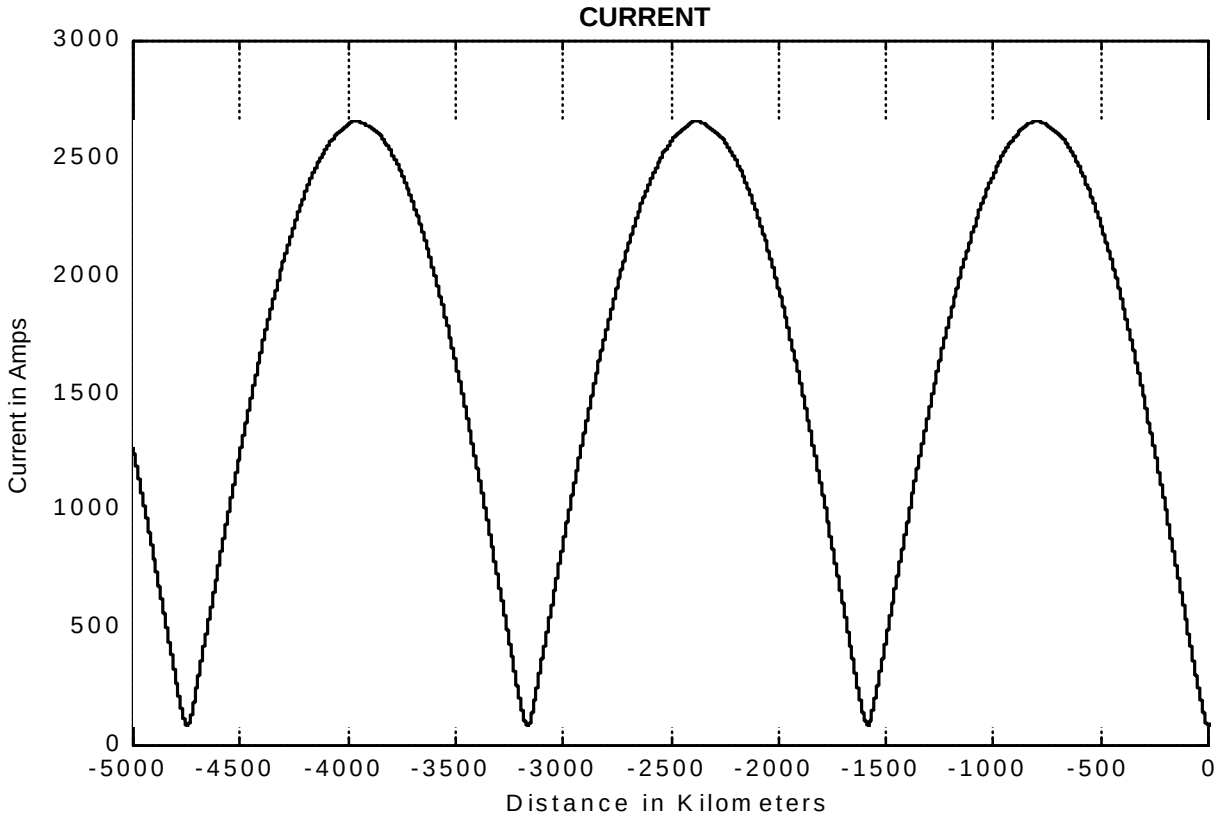
$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\frac{r_o}{r_i}}$	ε_0 = Permittivity of free space 8.85×10^{-12} [F/m]
$L = \frac{\mu_0}{2\pi}\ln\frac{r_o}{r_i}$	ε_r = Relative permittivity
$z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$	h = height of transmission line [m]
$\alpha = \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}$	r_o = outer radius of the conductor [m]
$\beta = \omega\sqrt{LC}$	r_i = inner radius of the conductor [m]
$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$	μ_0 = (mu) constant $4\pi\times10^{-7}$ [T·m/A]
$\mathbf{V}_S = \mathbf{V}_R\cosh\gamma d + z_0\mathbf{I}_R\sinh\gamma d$	α = attenuation constant
$\mathbf{I}_S = \frac{\mathbf{V}_R}{z_0}\sinh\gamma d + \mathbf{I}_R\cosh\gamma d$	β = phase constant [rad./m]
	γ = (gamma) propagation constant
	z_0 = surge impedance (has nothing to do with resistance) [Ω]
	R = resistance [Ω/m]
	L = inductance [H/m]
	G = conductance [v/m]
	C = capacitance [F/m]
	d = length of the line [m]
	$\mathbf{V}_S$ = voltage (complex) at the sending end [v]
	$\mathbf{V}_R$ = voltage (complex) at the receiving end [v]

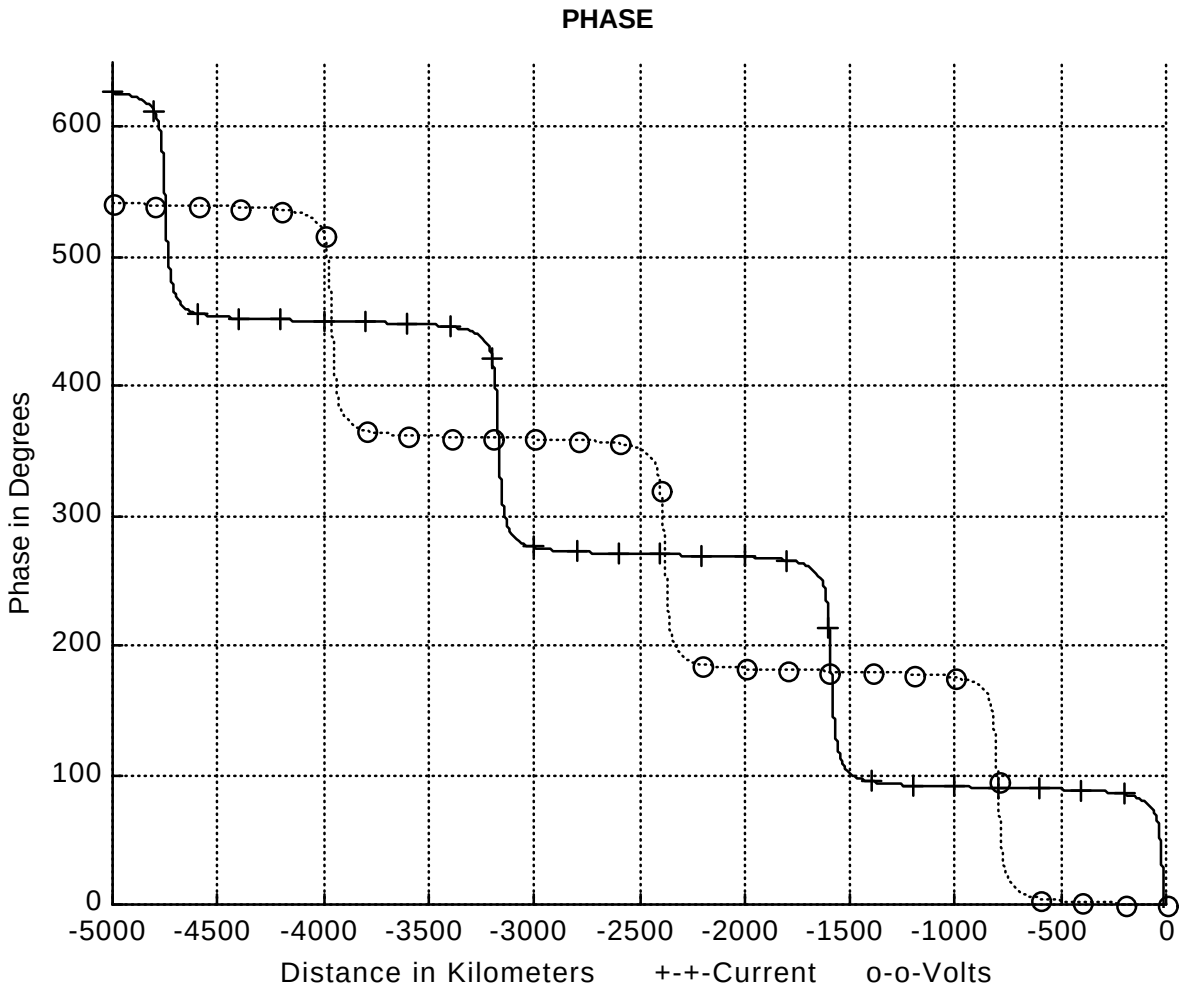
Program Output:

Omega = 376.9911
RadianPwrOut = -0.4510
DegreesPowerOutput = -25.8419
OutputCurrent = 90.0000 -43.5890i
C = 1.3996e-010
L = 1.9865e-007
Zo = 37.6741
Alpha = 0
Beta = 1.9878e-006
Gama = 0 +1.9878e-006i
Lambda = 3.1608e+006

Plots:


VOLTAGE
120
100
80
60
40
20
0
Kilovolts
-5000
-4500
-4000
-3500
-3000
-2500
-2000
-1500
-1000
-500
0
Distance in Kilometers


CURRENT
3000
2500
2000
1500
1000
500
0
Current in Amps
-5000
-4500
-4000
-3500
-3000
-2500
-2000
-1500
-1000
-500
0
Distance in Kilometers


PHASE
600
500
400
300
200
100
0
Phase in Degrees
-5000
-4500
-4000
-3500
-3000
-2500
-2000
-1500
-1000
-500
0
Distance in Kilometers
+-+-Current
o-o-Volts

Voltage and Current Data:

KILOMETERS	VOLTAGE (KW)	V. PHASE (°)	CURRENT (AMPS)	CURRENT (°)
0	100.00	0	100.00	-25.84
-100.00	98.36	0.39	489.46	79.62
-200.00	92.85	0.81	990.98	85.20
-300.00	83.68	1.30	1456.58	87.07
-400.00	71.23	1.95	1865.62	88.06
-500.00	55.99	2.91	2201.55	88.72
-600.00	38.57	4.69	2450.99	89.22
-700.00	19.74	9.73	2604.06	89.65
-800.00	3.40	95.13	2654.69	90.04
-900.00	20.32	170.62	2600.89	90.43
-1000.00	39.13	175.46	2444.79	90.85
-1100.00	56.49	177.19	2192.55	91.36
-1200.00	71.66	178.14	1854.19	92.02
-1300.00	84.01	178.78	1443.16	93.03
-1400.00	93.07	179.27	976.14	94.95
-1500.00	98.47	179.69	473.95	100.81
-1600.00	99.99	-179.92	107.96	-146.41
-1700.00	97.58	-179.53	589.19	-98.54
-1800.00	91.33	-179.10	1085.72	-94.31
-1900.00	81.49	-178.59	1541.69	-92.69
-2000.00	68.45	-177.90	1937.63	-91.79
-2100.00	52.73	-176.83	2257.57	-91.17
-2200.00	34.97	-174.75	2488.80	-90.69
-2300.00	15.98	-167.87	2622.16	-90.28
-2400.00	5.39	-38.89	2652.37	-89.89
-2500.00	24.07	-7.83	2578.24	-89.49
-2600.00	42.67	-4.09	2402.70	-89.05
-2700.00	59.65	-2.58	2132.70	-88.53
-2800.00	74.31	-1.72	1778.94	-87.82
-2900.00	86.06	-1.12	1355.55	-86.69
-3000.00	94.42	-0.65	879.79	-84.43
-3100.00	99.07	-0.24	374.31	-76.19
-3200.00	99.83	0.15	186.04	61.16
-3300.00	96.65	0.55	688.48	82.78
-3400.00	89.68	0.99	1178.87	86.11
-3500.00	79.17	1.53	1624.49	87.52
-3600.00	65.56	2.27	2006.70	88.35
-3700.00	49.39	3.46	2310.17	88.93
-3800.00	31.31	5.94	2522.84	89.40
-3900.00	12.24	16.00	2636.29	89.80
-4000.00	8.76	157.34	2646.04	90.19
-4100.00	27.80	173.30	2551.69	90.59
-4200.00	46.14	176.29	2356.99	91.04
-4300.00	62.73	177.62	2069.63	91.59
-4400.00	76.86	178.41	1701.02	92.36
-4500.00	87.98	178.98	1265.91	93.62
-4600.00	95.63	179.44	782.24	96.35
-4700.00	99.53	179.84	275.87	108.97
-4800.00	99.51	-179.77	280.58	-108.58
-4900.00	95.58	-179.37	786.99	-96.24
-5000.00	87.89	-178.91	1270.29	-93.54