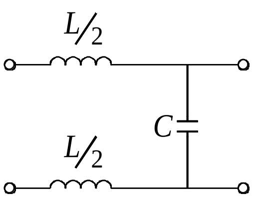
POWER TRANSMISSION AND DISTRIBUTION EE368

INDEX

THE LOSSLESS LINE

Most transmission lines fall into this category. The formulas are simplified since $R \to 0$ and $G \to \infty$.



γ = (gamma) propagation constant

α = attenuation constant

β = phase constant [rad/m]

A = volts

B = volts

Z_0 = characteristic or surge impedance [Ω]

$$\mathbf{V} = Ae^{gz} + Be^{-gz}$$

$$\mathbf{I} = -\frac{Ae^{\gamma z} - Be^{-\gamma z}}{Z_0}$$

when $R << \omega L$ and $G << \omega C$ then:

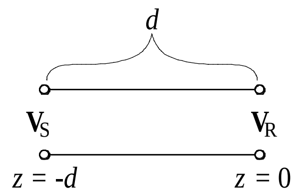
$$\alpha = \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}} \quad \text{and} \quad \beta = \omega\sqrt{LC}$$

$$\mathbf{V}_S = \mathbf{V}_R \cosh \beta d + Z_0 \mathbf{I}_R \sinh \beta d$$

$$\mathbf{I}_S = \frac{\mathbf{V}_R}{Z_0} \sinh \beta d + \mathbf{I}_R \cosh \beta d \qquad Z_0 = \sqrt{\frac{L}{C}}$$

TRANSMISSION LINE BASICS

Locations along the transmission line are traditionally referenced from the receiving end.



d = length [m]

$\mathbf{V}_S$ = voltage (complex) at the sending end [v]

$\mathbf{V}_R$ = voltage (complex) at the receiving end [v]

$\mathbf{I}_S$ = source current (complex) [A]

$\mathbf{I}_R$ = load current (complex) [A]

z = distance from the receiving end (a negative value or zero) [m]

γ = (gamma) propagation constant. Refer to "ALPHA, BETA, GAMMA" on page 3.

$$\mathbf{V}_R = A + B$$

$$\mathbf{I}_R = -\frac{1}{z_0}(A - B)$$

$$\mathbf{V}_S = \mathbf{V}_R \cosh \gamma d + z_0 \mathbf{I}_R \sinh \gamma d$$

$$\mathbf{I}_S = \frac{\mathbf{V}_R}{z_0} \sinh \gamma d + \mathbf{I}_R \cosh \gamma d$$

SHORT TRANSMISSION LINE

A transmission line is considered short when there is a small angular variation, $\beta z << 1$ radian.

TELEGRAPHER'S EQUATIONS

for a lossless line

$$\frac{\partial^2 v}{\partial z^2} = LC\frac{\partial^2 v}{\partial^2 t}$$

$$\frac{\partial^2 v}{\partial z^2} = LC\frac{\partial^2 i}{\partial^2 t}$$

L = inductance [H/m]

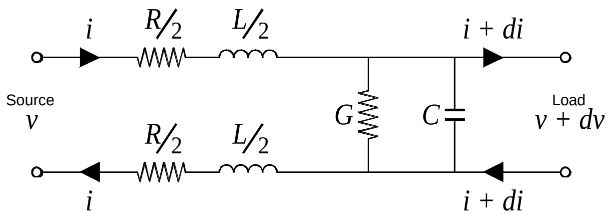
C = capacitance [F/m]

v = voltage [V]

z = distance along line [m]

t = time [s]

EQUIVALENT CIRCUIT FOR A TRANSMISSION LINE



KVL:

$$-v+\left(\frac{R}{2}dz\right)i+\left(\frac{L}{2}dz\right)\frac{di}{dt}+(v+dv)+\left(\frac{R}{2}dz\right)i+\left(\frac{L}{2}dz\right)\frac{di}{dt}=0$$

$$(R\,dz)i+(L\,dz)\frac{di}{dt}+dv=0$$

$$-dv=(R\,dz)i+(L\,dz)\frac{di}{dt}$$

$$\frac{dv}{dz}=-\left(Ri+L\frac{di}{dt}\right)$$

Phasor form: $\dfrac{\partial \mathbf{V}}{\partial z}=-(R+j\omega L)\mathbf{I}$

KCL:

$$-i+G\,dz(v+dv)+C\,dz\frac{d}{dt}(v+dv)+(i+di)=0$$

$$-di=G\,dz(v+dv)+C\,dz\frac{d}{dt}(v+dv)$$

$$\frac{di}{dz}=-Gv-C\frac{dv}{dt}$$

Phasor form: $\dfrac{\partial \mathbf{I}}{\partial z}=-(G+j\omega C)\mathbf{V}$

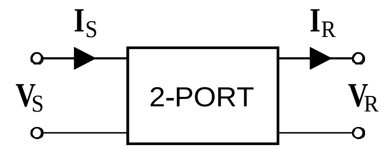
R = resistance [Ω/m]	i = current, amps [A]
L = inductance [H/m]	z = distance [m]
G = conductance [v/m]	$\mathbf{V}$ = V phasor
C = capacitance [F/m]	$\mathbf{I}$ = I phasor
	ω = phase angle [rad]

WAVELENGTH

$$\lambda=\frac{2\pi}{\beta}=\frac{2\pi}{\omega\sqrt{LC}}=\frac{2\pi v_p}{\omega}=\frac{v_p}{f}$$

- λ = wavelength [m]
- β = phase constant [rad./m]
- ω = frequency [rad./s]
- f = frequency [Hz]
- v_p = velocity of propagation (2.998×10^8 for a conductor in air) [m/s]

TWO-PORT SYSTEM



$$\begin{bmatrix}\mathbf{V}_S\\ \mathbf{I}_S\end{bmatrix}=\begin{bmatrix}\cosh(\gamma d) & Z_0\sinh(\gamma d)\\ \frac{1}{z_0}\sinh(\gamma d) & \cosh(\gamma d)\end{bmatrix}\times\begin{bmatrix}\mathbf{V}_R\\ \mathbf{I}_R\end{bmatrix}$$

This matrix equation is equivalent to:

$$\mathbf{V}_S=\cosh(\gamma d)\cdot\mathbf{V}_R+Z_0\sinh(\gamma d)\cdot\mathbf{I}_R \quad \text{and}$$

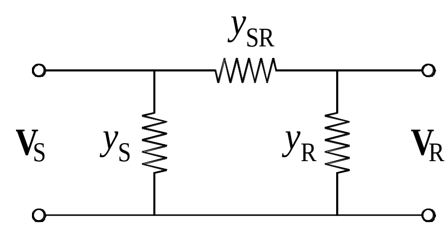
$$\mathbf{I}_S=\frac{1}{z_0}\sinh(\gamma d)\cdot\mathbf{V}_R+\cosh(\gamma d)\cdot\mathbf{I}_R$$

This can also be expressed:

$$\mathbf{V}_S=\mathbf{V}_R\left(\frac{e^{\gamma d}+e^{-\gamma d}}{2}\right)+z_0\mathbf{I}_R\left(\frac{e^{\gamma d}-e^{-\gamma d}}{2}\right)$$

$$\mathbf{I}_S=\frac{1}{z_0}\left[\mathbf{V}_R\left(\frac{e^{\gamma d}-e^{-\gamma d}}{2}\right)+Z_0\mathbf{I}_R\left(\frac{e^{\gamma d}+e^{-\gamma d}}{2}\right)\right]$$

THE PI EQUIVALENT MODEL



$$y_R=y_S=\frac{\cosh(\gamma d)-1}{z_0\sinh(\gamma d)}=\frac{\tanh\frac{\gamma d}{2}}{z_0}$$

$$y_{SR}=\frac{1}{z_0\sinh(\gamma d)}$$

OPEN-CIRCUIT COAXIAL LINE

$$\begin{bmatrix}\mathbf{V}_S\\ \mathbf{I}_S\end{bmatrix}=\begin{bmatrix}\cos bd & jZ_0\sin bd\\ \frac{j\sin bd}{Z_0} & \cos bd\end{bmatrix}\begin{bmatrix}\mathbf{V}_R\\ \mathbf{I}_R\end{bmatrix}$$

In this case, $\beta=3.33\times10^{-9}\,\omega\sqrt{\mu_r\varepsilon_r}$

VELOCITY OF PROPAGATION v_p

The speed at which a wave travels down the line. For a transmission line in air, this is near the speed of light, $c = 2.998 \times 10^8$ m/s

$$v_p = \frac{1}{\sqrt{LC}}$$

v_p = velocity of propagation [m/s]
L = inductance [H/m]
C = capacitance [F/m]

SURGE IMPEDANCE or CHARACTERISTIC IMPEDANCE

The cable materials and the arrangement of the conductors determine the surge impedance. It has nothing to do with resistance.

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Z_0 = surge impedance (has nothing to do with resistance) [Ω]
R = resistance [Ω/m]
L = inductance [H/m]
G = conductance [v/m]
C = capacitance [F/m]
ω = frequency [radians/sec.]

ALPHA, BETA, GAMMA

when $R << \omega L$ and $G << \omega C$ then:

$$\alpha = \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}$$

and $\beta = \omega\sqrt{LC}$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$
$$= \alpha + \mathbf{j}\beta$$
$$\mathbf{V} = Ae^{\gamma z} + Be^{-\gamma z}$$
$$\mathbf{I} = -\frac{Ae^{\gamma z} - Be^{-\gamma z}}{Z_0}$$
$$V_R = A + B$$
$$I_R = -\frac{1}{Z_0}(A - B)$$

α = attenuation constant
β = phase constant [rad./m]
γ = (gamma) propagation constant
Z_0 = surge impedance (has nothing to do with resistance) [Ω]
A = volts
B = volts
V_R = voltage at the receiving end
I_R = current at the receiving end

CAPACITANCE PER UNIT LENGTH

Single line in air (capacitance decreases with height):

$$C = \frac{2\pi\varepsilon_0}{\ln\frac{2h}{r}}$$

Two conductors in a **bundle**:

$$C = \frac{2\pi\varepsilon_0}{\ln\frac{\sqrt{D_{aai}\,D_{abi}}}{\sqrt{r\,D_{ab}}}}$$

Coaxial cable:

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\frac{r_0}{r_i}}$$

ε_0 = Permittivity of free space 8.85×10^{-12} [F/m]
ε_r = relative permittivity [constant]
h = height of transmission line [m]
r = radius of the conductor [m]
D_{aai} = distance from conductor a to its image [m]
D_{abi} = distance from conductor a to the image of conductor b [m]
D_{ab} = distance from conductor a to conductor b [m]
r_0 = outer radius of a coaxial conductor [m]
r_i = inner radius of a coaxial conductor [m]

POS./NEG.-SEQUENCE CAPACITANCE

3-phase positive or negative sequence capacitance:

$$C_{+/-} = \frac{2\pi\varepsilon_0}{\ln\frac{GMD_{+/-}}{GMR_{+/-}}}$$

where the **geometric mean distance** between conductors is:

$$GMD_{+/-} = \sqrt[3]{D_{ab}D_{ac}D_{bc}}$$

and the **geometric mean radius** is:

$$GMR_{+/-} = \sqrt[3]{r_a r_b r_c}$$

ZERO-SEQUENCE CAPACITANCE

3-phase zero-sequence capacitance:

$$C_0 = \frac{2\pi\varepsilon_0}{3\ln\frac{GMD_0}{GMR_0}}$$

where the **geometric mean distance** between conductors is:

$$GMD_0 = \sqrt[9]{D_{aai}D_{abi}D_{aci}D_{bai}D_{bbi}D_{bci}D_{cai}D_{cbi}D_{cci}}$$

and the **geometric mean radius** is:

$$GMR_0 = \sqrt[9]{r_a r_b r_c D_{ab}D_{ac}D_{ba}D_{bc}D_{ca}D_{cb}}$$

INDUCTANCE PER UNIT LENGTH

Single line in air (inductance increases with height):

$$L = \frac{\mu_0}{2\pi} \ln \frac{2h}{r}$$

Coaxial cable:

$$L = \frac{\mu_0 \mu_r}{2\pi} \ln \frac{r_0}{r_i}$$

μ_0 = (mu) Permeability constant $4\pi \times 10^{-7}$ [H/m, T·m/A]

μ_r = (mu) relative permeability, a value near 1 for many materials

h = height of transmission line [m]

r = radius of the conductor [m]

r_0 = outer radius of a coaxial conductor [m]

r_i = inner radius of a coaxial conductor [m]

POS./NEG.-SEQUENCE INDUCTANCE

3-phase positive or negative sequence inductance:

$$\boxed{L_{+/-} = \frac{\mu_0}{2\pi} \ln \frac{GMD_{+/-}}{GMR_{+/-}}}$$

where the **geometric mean distance** between conductors is:

$$GMD_{+/-} = \sqrt[3]{D_{ab} D_{ac} D_{bc}}$$

and the **geometric mean radius** is:

$$GMR_{+/-} = \sqrt[3]{r_a r_b r_c}$$ (see note)

Note: Apply a multiplier of $e^{-1/4}$ to the *physical* radius of each conductor.

ZERO-SEQUENCE INDUCTANCE

3-phase zero-sequence inductance:

$$\boxed{L_0 = \frac{\mu_0}{2\pi} 3 \ln \frac{GMD_0}{GMR_0}}$$

where the **geometric mean distance** between conductors is:

$$GMD_0 = \sqrt[9]{D_{aai} D_{abi} D_{aci} D_{bai} D_{bbi} D_{bci} D_{cai} D_{cbi} D_{cci}}$$

and the **geometric mean radius** is:

$$GMR_0 = \sqrt[9]{r_a r_b r_c D_{ab} D_{ac} D_{ba} D_{bc} D_{ca} D_{cb}}$$ (see note)

Note: Apply a multiplier of $e^{-1/4}$ to the *physical* radius of each conductor.

d_c COMPLEX DEPTH

Complex depth is an adustment to the actual depth of the conductor image, used when calculating **inductance**.

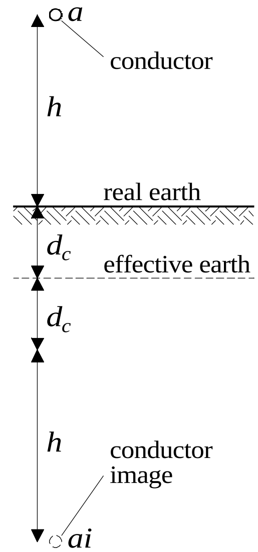
The value of d_c added to the above-ground height of the conductors gives the distance to effective earth. In other words, instead of $D_{aai} = 2h$, we now have $D_{aai} = 2h + 2d_c$.

$$d_c = \frac{1}{(1+j)\sqrt{\pi f \mu_0 \sigma}}$$

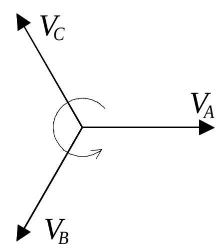
f = frequency [Hz]

μ_0 = (mu) Permeability constant $4\pi \times 10^{-7}$ [H/m, T·m/A]

σ = constant, 0.01 for limestone [(Ω-m)$^{-1}$]



POSITIVE SEQUENCE



$V_a = V_{ag} \angle 0$

$V_b = V_{ag} \angle -120$

$V_c = V_{ag} \angle 120$

STANDING WAVE RATIO

the ratio of peak voltage to minimum voltage:

$$\mathrm{SWR} = \frac{\mathrm{MAX}(V(z)_{rms})}{\mathrm{MIN}(V(z)_{rms})} = \frac{1+\rho}{1-\rho}$$

where ρ is the magnitude of the reflection coefficient

TRAVELING WAVES

$$v(t,z) = \underbrace{F_1\left(t - z\sqrt{LC}\right)}_{\text{forward wave } V^+} + \underbrace{F_2\left(t + z\sqrt{LC}\right)}_{\text{reverse wave } V^-}$$

$$i(t,z) = \underbrace{\frac{F_1}{z_0}\left(t - z\sqrt{LC}\right)}_{\text{forward wave } I^+} - \underbrace{\frac{F_2}{z_0}\left(t + z\sqrt{LC}\right)}_{\text{reverse wave } I^-}$$

Forward traveling wave: $e^{-\gamma z}$

Reverse traveling wave: $e^{+\gamma z}$

TRANSIT TIME

$$t_d = \frac{d}{v_p}$$

t_d = 1-way transit time [s]
d = length of transmission line [m]
v_p = velocity of propagation [m/s]

VOLTAGE DROP ACROSS INLINE ELEMENTS

Series inductance: $v = \mathbf{I}(j\omega L)$

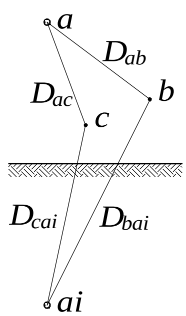
Series capacitance: $v = \dfrac{\mathbf{I}}{j\omega C}$

PEAK, RMS, and VOLTAGE TO GROUND

$$V_{ab\,peak} = V_{ab\,rms}\sqrt{2} \qquad V_{ag} = \frac{V_{ab(3\phi)}}{\sqrt{3}}$$

VOLTAGE BETWEEN TWO POINTS *b* and *c* DUE TO A CHARGED LINE *a*

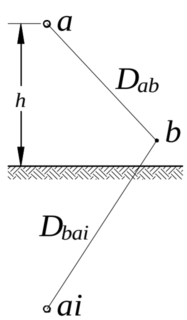
$$V_{bc} = \frac{q_a}{2\pi\varepsilon_0}\ln\frac{D_{ab}D_{cai}}{D_{ac}D_{bai}}$$



$q_a = CV$ = unit charge on the line [c/m]
ε_0 = Permittivity of free space 8.85×10^{-12} [F/m]
D_{ab} = distance from line *a* to point *b* [m]
D_{cai} = distance from point *c* to the image of line *a* [m]

VOLTAGE TO GROUND AT POINT b DUE TO TRANSMISSION LINE a

$$V_{bg} = V_a \frac{\ln\dfrac{D_{bai}}{D_{ab}}}{\ln\dfrac{2h}{r_a}}$$



D_{bai} = distance from point *b* to the image of line *a* [m]
D_{ab} = distance from line *a* to point *b* [m]
r = radius of conductor *a* [m]

CAPACITANCE MATRIX

The upper and lower triangles of the capacitance matrix are equal.

$$\begin{bmatrix} q_a \\ q_b \\ q_c \end{bmatrix} = \begin{bmatrix} C_{aa} & C_{ab} & C_{ac} \\ C_{ab} & C_{bb} & C_{bc} \\ C_{ac} & C_{bc} & C_{cc} \end{bmatrix}\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix}$$

$$C_{aa} = \frac{2\pi\varepsilon_0}{\ln\dfrac{2h}{r_a}} \qquad C_{ab} = \frac{2\pi\varepsilon_0}{\ln\dfrac{D_{abi}}{D_{ab}}}$$

ELECTRIC SHOCK

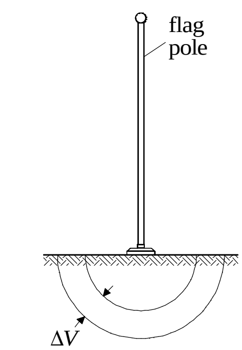
Danzeil's Electrocution Formula:

$$I = \frac{0.165}{\sqrt{t}}A$$

Perception level: 1 mA
Let go level: 10-20 mA
Death level: 100 mA
Body resistance: 1kΩ

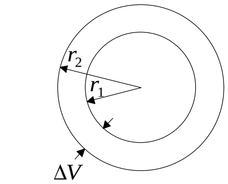
STEP VOLTAGE

Step voltage is the potential per unit length across the surface of the earth. This can be a shock hazard in the case of a lightning strike or large fault (short circuit).



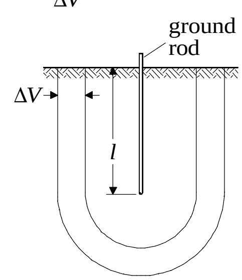
For the flag pole:

$$\Delta V = \frac{I}{2\pi\sigma}\left(\frac{r_2 - r_1}{r_1 r_2}\right)$$



For the ground rod:

$$\Delta V = \frac{I}{2\pi\sigma l}\ln\frac{r_2(l + r_1)}{r_1(l + r_2)}$$

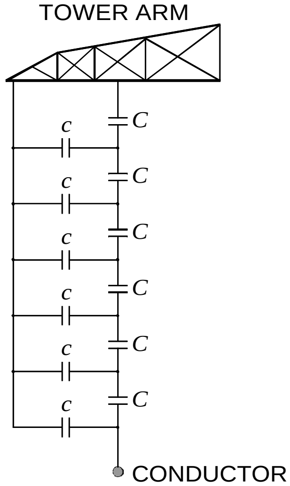


ΔV = step voltage (as shown) [V]
σ = constant, 0.01 for limestone $[(\Omega\text{-m})^{-1}]$
l = length of the ground rod [m]

VOLTAGE ACROSS INSULATORS DUE TO CAPACITANCE

$$V_n = V_g \frac{\sinh \alpha n}{\sinh \alpha z}$$

$$\alpha = \sqrt{c / C}$$



α = capacitance ratio

c = capacitance between insulator and arm [F]

C = capacitance across insulator [F]

V_n = voltage between arm and insulator unit n [V]

V_g = line voltage [V]

n = integer value denoting a particular insulator unit. n = 1 is the unit attached at the tower arm

z = total no. of insulator units

ELECTRIC FIELD

Perception level: 10 kV/m rms

Annoyance level (sparks): 15-20 kV/m rms

Design limit (peak): 2200 kV/m or 22 kV/cm

Critical (air breakdown): 3000 kV/m or 30 kV/cm

$$\mathbf{E}_r = \frac{q_l}{2\pi\varepsilon_0 r}\hat{\mathbf{a}}_r$$

Breakdown in dry air:

$$E_{BK} = 3000\left(\frac{17.9P}{459+T}\right)^{2/3}$$

$\mathbf{E}_r$ = radial electric field [V/m]

q_l = line charge [C/m]

r = conductor radius [m]

$\hat{\mathbf{a}}_r$ = radial unit vector

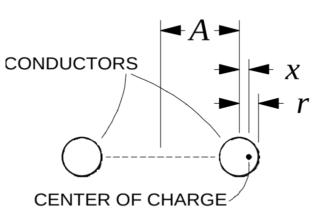
E_{BK} = breakdown [V/m]

P = atmospheric pressure [in. Hg]

T = temperature [°F]

Electric field for a bundle of 2:

$$E = \frac{q_l / 2}{2\pi\varepsilon_0 (r-x)} + \frac{q_l / 2}{2\pi\varepsilon_0 (2A + r + x)}$$



x = displacement of the center of charge from the center of the conductor [m]

A = bundle radius, measured from center of bundle to center of conductor [m]

MARKT MENGELE METHOD

for computing average maximum peak bundle gradient—used in noise calculations

1. Treat each phase bundle as a single equivalent conductor with radius: $r_{eq} = \left(NrA^{N-1}\right)^{1/N}$
2. Find the C_{NxN} matrix. Kron reduce it to C_{3x3}. Select the phase bundle with the maximum diagonal C term (this is usually the inside bundle). Put V_{max} on it, ($-V_{max}/2$) on the other two bundles, and compute the peak bundle line charge $q_{l\ peak}$.
3. Assuming equal charge division, calculate the average maximum bundle gradient and the average maximum peak bundle gradient.

$$E_{avg\ \max} = \frac{q_{l\ peak}}{2pNe_0 r}$$

$$E_{avg\ peak} = E_{avg\ \max}\left[1 + (N-1)\frac{r}{A}\right]$$

N = number of conductors in the bundle

r = conductor radius (not the bundle radius) [m]

A = bundle radius, measured from center of bundle to center of conductor [m]

NOISE AND INTERFERENCE

TRANSMISSION LINE NOISE

Transmission line noise is caused by corona. It has a 120 Hz base frequency and is the effect of positive and negative ions moving back and forth. The attenuation is 3 dB per doubling distance from the line. This is a slow attenuation due to the length of the line. In the 1000 kV range, sound is a limiting factor.

$$\text{sound level (dB)} = 20\log_{10}\frac{\text{sound pressure}}{\text{20 nPa reference pressure}}$$

Noise per phase:

$$AN = 120\log g + K\log N + 55\log d - 11.4\log D + AN_0$$

AN = some kind of noise [dB or dBA?, meaning above level of perception]

g = peak surface gradient by Markt Mengele method [kV/cm]

d = dunno [m]

D = wire to listener distance [m]

AN_0 = some other kind of noise [dB]

RADIO INTERFERENCE

Corona discharge – occurs only on very high voltage lines.

Gap discharge – usually indicates a physical problem, can occur on distribution lines.

GEOMAGNETIC STORM

Low frequency flux, almost DC. Occurs in east/west lines in polar areas.

ELECTRIC FIELD and CAPACITANCE

in a coaxial conductor

** $\mathbf{E}_r = \dfrac{q_l}{2pe_0 e_r r}$

$$V = \int_{r=r_i}^{r_o} \mathbf{E}_r \, dr$$

$$= \frac{q_l}{2pe_0 e_r} \int_{r=r_i}^{r_o} \frac{1}{r} dr$$

$$= \frac{q_l}{2pe_0 e_r} \ln \frac{r_o}{r_i}$$

$$C = \frac{q_l}{V} = \frac{2pe_0 e_r}{\ln \frac{r_o}{r_i}}$$

$$E_{\max} = \frac{q_l}{2pe_0 e_r r_i} = \frac{V}{r_i \ln \frac{r_o}{r_i}}$$

$\mathbf{E}_r$ = radial electric field [V/m]
q_l = line charge [C/m]
ε_0 = Permittivity of free space 8.85×10^{-12} [F/m]
ε_r = relative permittivity [constant]
r = radial distance [m]
r_i = inner conductor radius [m]
r_o = outer conductor radius [m]
V = voltage between inner and outer conductor [V]
C = capacitance per meter [F/m]
E_{max} = maximum electric field (near center conductor) [m]

**When using this formula to find the height above ground at which breakdown occurs, *r* is the conductor radius, not the height, because breakdown begins at the surface of the conductor.

MAGNETIC FLUX

MAGNETIC FLUX

$$L = \frac{N\Phi}{I}$$

$$H_\phi = \frac{1}{2\pi r}$$

$$\mathbf{B} = \mu \mathbf{H}$$

$$\Phi_S = \oint_s \mathbf{B} \cdot d\mathbf{s}$$

L = inductance [H]
N = number of turns
I = current [A]
H_ϕ = magnetic field intensity (direction by right-hand rule) [A/m]
$\mathbf{B}$ = magnetic flux density [W/m^2]
μ = permiability of free space $4\pi\times10^{-7}$
ε_r = relative permittivity [constant]
r = radial distance [m]
Φ_S = amount of magnetic flux passing through a surface [H/m^2]

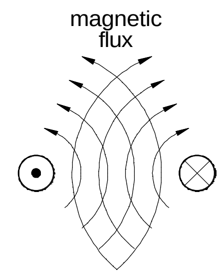
FLUX LINKING – 2 conductors

The amount of flux linking two wires is the amount of flux passing between them. This applies to two conductors of equal radius carrying equal current in opposite directions.

$$\Phi = \int_{x=r_{eq}}^{D-r_{eq}} \frac{\mu_0 I}{2\pi x} dx + \int_{x=r_{eq}}^{D-r_{eq}} \frac{\mu_0 I}{2\pi x} dx = \frac{\mu_0 I}{\pi} \ln \frac{D - r_{eq}}{r_{eq}}$$

$$L_l = \frac{\mu_0}{\pi} \ln \frac{D}{r_{eq}} \qquad r_{eq} = re^{-1/4} = 0.7788r$$

D = distance between two conductors (center to center) [m]
I = current [A]
x = a distance along D [m]
Φ = amount of magnetic flux passing through a surface [H/m^2]
L_l = inductance per meter between 2 conductors [H/m]
r_{eq} = equivalent radius [m]



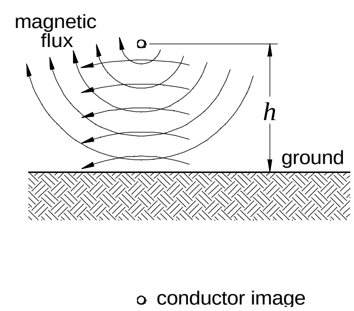
FLUX LINKING – 1 conductor above earth

The amount of flux linking a single wire above earth is the total flux passing between the conductor and the ground, summing the contributions by the conductor and by its image.

$$\Phi = \frac{\mu_0 I}{2\pi}\left[\int_{x=r_{eq}}^{h} \frac{1}{x} dx + \int_{x=h}^{2h-r_{eq}} \frac{1}{x} dx\right] = \frac{\mu_0 I}{2\pi} \ln \frac{2h - r_{eq}}{r_{eq}}$$

NOTE: Use the equivalent radius, $r_{eq} = re^{-1/4}$.

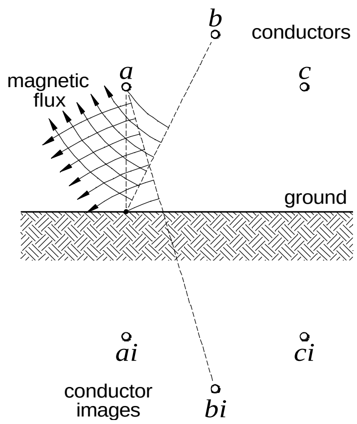
$$L_l = \frac{\mu_0}{2\pi} \ln \frac{2h - r_{eq}}{r_{eq}}$$



FLUX LINKING TO GROUND

multiple conductors above earth

The diagram and formula concern the flux linking for conductor a due to conductor b only.



$$\Phi_{ba} = \frac{\mu_0 I_b}{2\pi}\int_{r=D_{ba}}^{D_{bg}} \frac{dr}{r} + \frac{\mu_0 I_b}{2\pi}\int_{r=D_{gbi}}^{D_{abi}} \frac{dr}{r} = \frac{\mu_0 I_b}{2\pi}\ln\frac{D_{abi}}{D_{ab}}$$

P MATRIX

The relationship between three conductors in air is:

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \frac{1}{2\pi\varepsilon_0}\begin{bmatrix} P_{aa} & P_{ab} & P_{ac} \\ P_{ba} & P_{bb} & P_{bc} \\ P_{ca} & P_{cb} & P_{cc} \end{bmatrix}\begin{bmatrix} q_a \\ q_b \\ q_c \end{bmatrix}$$

where V_{ag} is the voltage from conductor *a* to ground,

where $P_{aa} = \ln\frac{D_{aai}}{r_a}$ with D_{aai} being the distance between conductor a and its image, and r_a being the radius of conductor a.

where $P_{ab} = \ln\frac{D_{abi}}{D_{ab}}$ with D_{abi} being the distance between conductor a and the image of conductor b, and D_{ab} being the distance between conductors *a* and b. The remaining P terms follow this pattern.
The expression can also be written:

$$V_{abc} = \frac{1}{2\pi\varepsilon_0} P_{abc} Q_{abc}$$

Relationships involving capacitance are:

$$2\pi\varepsilon_0 \times P_{abc}{}^{-1*} = C_{abc} \quad \text{and} \quad C_{abc} \times V_{abc} = Q_{abc}$$

The term $P_{abc}{}^{-1*}$ means a matrix in which the inverse of the individual members has been carried out.

WITH GROUND CONDUCTORS:

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \\ V_{vg} \\ V_{wg} \end{bmatrix} = \frac{1}{2\pi\varepsilon_0}\begin{bmatrix} P_{aa} & P_{ab} & P_{ac} & P_{av} & P_{aw} \\ P_{ba} & P_{bb} & P_{bc} & P_{bv} & P_{bw} \\ P_{ca} & P_{cb} & P_{cc} & P_{cv} & P_{cw} \\ P_{va} & P_{vb} & P_{vc} & P_{vv} & P_{vw} \\ P_{wa} & P_{wb} & P_{wc} & P_{wv} & P_{ww} \end{bmatrix}\begin{bmatrix} q_a \\ q_b \\ q_c \\ q_v \\ q_w \end{bmatrix}$$

Since the grounds *v* and *w* have zero potential, the matrix dimension 5 can be reduced to 3.

EQUIVALENT BUNDLE RADIUS

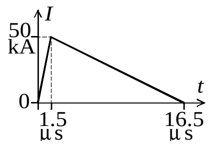
$$r_{eq} = \left(NrA^{N-1}\right)^{1/N}$$

r_{eq} = equivalent bundle radius [m]
N = number of conductors in the bundle
r = conductor radius [m]
A = bundle radius, measured from center of bundle to center of conductor [m]

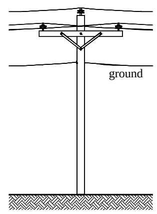
LIGHTNING AND TRANSIENTS

LIGHTNING

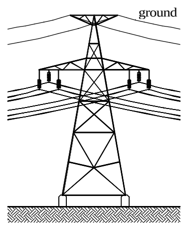
Lightning is essentially a current souce. The model at right approximates the waveform of a 1.5 × 50 strike, meaning 1.5 µs rise time, 50 kA peak. Fall time is assumed 10× the rise time.



On **distribution lines**, chances are greater that lightning damage will be from a ground return stroke, so the ground wire is placed below the current-carrying conductors.



With **transmission lines**, the ground wire(s) is placed above the current-carrying conductors, providing a *shadow angle* of protection.



GROUND ROD RESISTANCE

$$R = \frac{1}{2\pi\sigma h}\ln\left(\frac{2h}{a} - 1\right)$$

R = ground rod resistance [Ω]
σ = conductivity of earth [1/(Ω ·m)]
h = rod depth [m]
a = rod radius [m]

$\mathbf{r}_V$ REFLECTION COEFFICIENT OF VOLTAGE

The reflection coefficient is the factor by which the voltage is multiplied to find the voltage of the reflected wave. A voltage is reflected when it reaches a discontinuity in the line, such as a load, tap, or connection to a line of different characteristic impedance. The reflection coefficient for an open-ended line is 1 and for a short it is –1.

$$r_V = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Z_L = load impedance [Ω]
Z_0 = characteristic impedance of the line [Ω]

$\mathbf{t}_V$ TRANSMISSION COEFFICIENT OF VOLTAGE

$$\tau_V = 1 + \rho_V = \frac{2Z_L}{Z_L + Z_0}$$

ρ_V = reflection coefficient of voltage
Z_L = load impedance [Ω]
Z_0 = characteristic impedance of the line [Ω]

$\mathbf{r}_I$ REFLECTION COEFFICIENT OF CURRENT

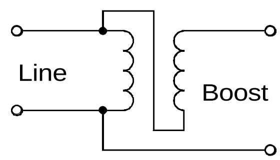
$$\rho_I = \frac{I^-}{I^+} = -\rho_V = \frac{-(Z_L - Z_0)}{Z_L + Z_0}$$

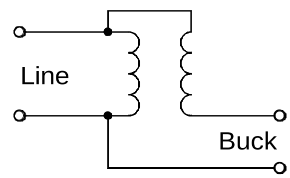
I^+ = forward-traveling current wave [A]
I^- = reverse-traveling current wave [A]
ρ_V = reflection coefficient of voltage
Z_L = load impedance [Ω]
Z_0 = characteristic impedance of the line [Ω]

TRANSFORMERS

BOOST AND BUCK

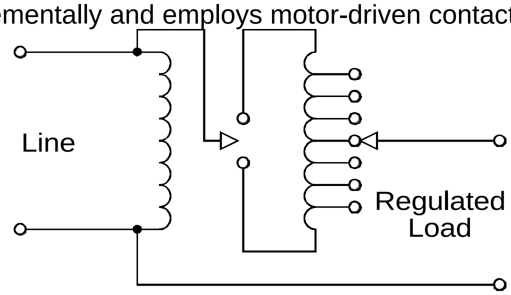
These two transformer types are the basis for the regulating transformer below.





VOLTAGE REGULATOR

The voltage regulator is used at substations to maintain a near constant output voltage under varying load conditions. The transformer can boost or buck incrementally and employs motor-driven contacts.



A substation will typically have two of these units. **Substation capacity** is limited to 75% load. If one transformer should fail, the remaining unit can handle 150% of its rated load for 2 hours—enough time to switch loads to other substations.

MISCELLANEOUS

Corona is the ionization of air; it is a source of radio interference, power loss, hissing, crackling noise, and visible blue light. Not a catastrophic arc. The effect is intensified when moisture is in the air.

A **flashover** is an arc.

Sparking occurs with poor, dirty connections.

A **trickle pulse** is thousands of pulses per half-cycle, responsible for interference in the AM band.

Reasons for using **multiple conductors**: Most current flows in the outer ½ of the conductor, especially at higher frequencies. The use of multiple conductors reduces the electromagnetic field, when arranged in a circular pattern, but even two conductors side-by-side has a dramatic effect.

Surge impedance loading means that $z_{LOAD} = z_0$, i.e. the impedance of the load is the same as the characteristic impedance of the line. Under this condition, no wave reflection occurs.

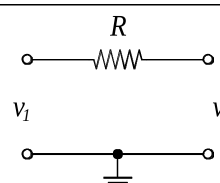
Charge on a conductor. The only way a conductor can have a charge is by direct contact. No charge does not mean no voltage.

GRAPHING TERMINOLOGY

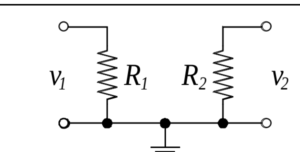
With x being the horizontal axis and y the vertical, we have a graph of y **versus** x or y **as a function of** x. The x-axis represents the **independent variable** and the y-axis represents the **dependent variable**, so that when a graph is used to illustrate data, the data of regular interval (often this is time) is plotted on the x-axis and the corresponding data is dependent on those values and is plotted on the y-axis.

MILLER THEOREM

The circuit at left may be replaced by the circuit at right. One of the resistances in the circuit at right will actually be an admittance.

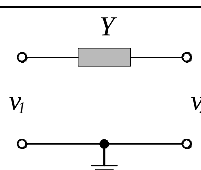


$$K = v_2 / v_1$$

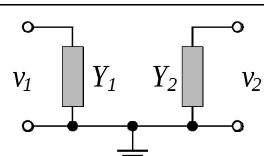


$$R_1 = \frac{R}{1-K} \quad R_2 = \frac{R}{1-1/K}$$

For an admittance Y we have:



$$K = v_2 / v_1$$



$$Y_1 = Y(1-K)$$
$$Y_2 = Y(1-1/K)$$

DOT PRODUCT

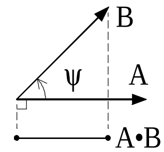
The dot product is a scalar value.

$$\mathbf{A}\bullet\mathbf{B} = (\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z)\bullet(\hat{\mathbf{x}}B_x + \hat{\mathbf{y}}B_y + \hat{\mathbf{z}}B_z) = A_xB_x + A_yB_y + A_zB_z$$

$$\mathbf{A}\bullet\mathbf{B} = |\mathbf{A}||\mathbf{B}|\cos\psi_{AB}$$

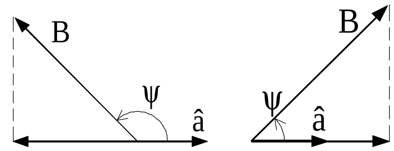
$$\hat{\mathbf{x}}\bullet\hat{\mathbf{y}} = 0, \quad \hat{\mathbf{x}}\bullet\hat{\mathbf{x}} = 1$$

$$\mathbf{B}\bullet\hat{\mathbf{y}} = (\hat{\mathbf{x}}B_x + \hat{\mathbf{y}}B_y + \hat{\mathbf{z}}B_z)\bullet\hat{\mathbf{y}} = B_y$$



Projection of **B** along **â**:

$$(\mathbf{B}\bullet\hat{\mathbf{a}})\hat{\mathbf{a}}$$



The dot product is **commutative** and **distributive**:

$$\mathbf{A}\bullet\mathbf{B} = \mathbf{B}\bullet\mathbf{A} \qquad \mathbf{A}\bullet(\mathbf{B}+\mathbf{C}) = \mathbf{A}\bullet\mathbf{B} + \mathbf{A}\bullet\mathbf{C}$$

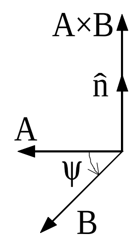
CROSS PRODUCT

$$\mathbf{A}\times\mathbf{B} = (\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z)\times(\hat{\mathbf{x}}B_x + \hat{\mathbf{y}}B_y + \hat{\mathbf{z}}B_z)$$
$$= \hat{\mathbf{x}}(A_yB_z - A_zB_y) + \hat{\mathbf{y}}(A_zB_x - A_xB_z) + \hat{\mathbf{z}}(A_xB_y - A_yB_x)$$

$$\mathbf{A}\times\mathbf{B} = \hat{\mathbf{n}}|\mathbf{A}||\mathbf{B}|\sin\psi_{AB}$$

where $\hat{\mathbf{n}}$ is the unit vector normal to both **A** and **B** (thumb of right-hand rule).

$$\mathbf{B}\times\mathbf{A} = -\mathbf{A}\times\mathbf{B}$$



The cross product is **distributive:**

$$\mathbf{A}\times(\mathbf{B}+\mathbf{C}) = \mathbf{A}\times\mathbf{B} + \mathbf{A}\times\mathbf{C}$$

Ñ NABLA, DEL OR GRAD OPERATOR

$$\nabla \equiv \hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z}$$

GEOMETRY

SPHERE

Area $A = 4\pi r^2$

Volume $V = \frac{4}{3}\pi r^3$

ELLIPSE

Area $A = \pi AB$

Circumference

$$L \approx 2\pi\sqrt{\frac{a^2+b^2}{2}}$$

TRIGONOMETRIC IDENTITIES

$\gamma = \alpha + \mathbf{j}\beta$

$\sinh(\gamma d) = \sinh(\alpha d)\cos(\beta d) + j\cosh(\alpha d)\sin(\beta d)$

$\cosh(\gamma d) = \cosh(\alpha d)\cos(\beta d) + j\sinh(\alpha d)\sin(\beta d)$

$$\tanh\left(\frac{\gamma d}{2}\right) = \frac{\sinh(\alpha d) + j\sin(\beta d)}{\cosh(\alpha d) + \cos(\beta d)}$$

$$\tanh\left(\frac{\gamma d}{2}\right) = \frac{\cosh(\partial d) - 1}{\sinh(\partial d)}$$

$$\sin(\beta d) = (\beta d) - \frac{(\beta d)^3}{3!} + \cdots$$

$$\cos(\beta d) = 1 - \frac{(\beta d)^2}{2!} + \cdots$$

$$\sinh(\alpha d) = (\alpha d) + \frac{(\alpha d)^3}{3!} + \cdots$$

$$\cosh(\alpha d) = 1 + \frac{(\alpha d)^2}{2!} + \cdots$$

CONSTANTS

Avogadro's number [molecules/mole]	$N_A = 6.02 \times 10^{23}$
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ J/K
	$= 8.62 \times 10^{-5}$ eV/K
Elementary charge	$q = 1.60 \times 10^{-19}$ C
Electron mass	$m_0 = 9.11 \times 10^{-31}$ kg
Permittivity of free space	$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m
Permeability constant (mu)	$\mu_0 = 4\pi \times 10^{-7}$ [H/m]
Planck's constant	$h = 6.63 \times 10^{-34}$ J-s
	$= 4.14 \times 10^{-15}$ eV-s
Rydberg constant	$R = 109{,}678$ cm^{-1}
kT @ room temperature	$kT = 0.0259$ eV
Speed of light	$c = 2.998 \times 10^8$ m/s
1 Å (angstrom)	10^{-8} cm = 10^{-10} m
1 μm (micron)	10^{-4} cm

1 nm = 10Å = 10^{-7} cm

1 eV = 1.6 × 10^{-19} J

1 V = 1 J/C 1 N/C = 1 V/m 1 J = 1 N·m = 1 C·V

CALCULUS OPERATIONS

$V_r = \int \mathbf{E}_r \, dr$

$\frac{dv}{dz} = E$?

$v_L(t) = L\frac{di}{dt}$

$i_L(t) = \frac{1}{L}\int_0^t v \, d\tau + i_0$

$v_C(t) = \frac{1}{C}\int_0^t i \, d\tau + v_0$

$i_C(t) = C\frac{dv}{dt}$

V_r = voltage [V]
$\mathbf{E}_r$ = radial electric field [m]
i_L = current in an inductor [A]
i_C = current in a capacitor [A]
v_L = voltage across an inductor [V]
v_C = voltage across a capacitor [V]

PROPERTIES OF THE NATURAL LOG

$\ln A + \ln B = \ln AB$

$\ln A - \ln B = \ln \frac{A}{B}$

$A \ln B = \ln B^A$

$\ln A = B \rightarrow e^B = A$

$e^{A \ln B} = B^A$

$\ln e^A = A$