APPLIED THERMODYNAMICS ME 320

INDEX

BASIC THERMODYNAMICS

0TH LAW OF THERMODYNAMICS

Two bodies which are each in thermal equilibrium with a third body are in thermal equilibrium with each other.

1ST LAW OF THERMODYNAMICS

The Conservation of Energy Principle

The amount of energy gained by a system is equal to the amount of energy lost by the surroundings.

2ND LAW OF THERMODYNAMICS

Processes occur in a certain direction and energy has quality as well as quantity. For example, heat flows from a high temperature place to a low temperature place, not the reverse. Another example, electricity flowing through a resistive wire generates heat, but heating a resistive wire does not generate electricity.

Kelvin-Planck statement: It is impossible for any device that operates on a cycle to receive heat from a single reservoir and produce a net amount of work.

Clausius statement: It is impossible to construct a device that operates in a cycle and produces no effect other than the transfer of heat from a lower temperature body to a higher temperature body.

STATE POSTULATE

The state of a simple compressible system is completely specified by two independent, intensive properties. Two properties are **independent** if one property can be varied while the other one is held constant. Properties are **intensive** if they do not depend on size, e.g. the properties of temperature, pressure, entropy, density, specific volume.

UNITS

Energy, work, heat transfer: [J]

J (joule) = $N\cdot m = V\cdot C = W\cdot s = A\cdot V\cdot s = F\cdot V^2 = \dfrac{C^2}{F}$

1 kJ = 0.94782 Btu   1 Btu = 1.055056 kJ

Rate of energy, work or heat transfer: [J/s or W]

W (watt) = $\dfrac{J}{s} = \dfrac{N\cdot m}{s} = \dfrac{C\cdot V}{s} = V\cdot A = \dfrac{F\cdot V^2}{s} = \dfrac{1}{746}HP$

Pressure: [Pa or N/m^2 or kg/ms^2]

Pa (pascal) = $\dfrac{N}{m^2} = \dfrac{kg}{m\cdot s^2} = \dfrac{J}{m^3} = \dfrac{W\cdot s}{m^3}$ = 1.45038×10^{-4} psi

Density: [kg/m^3]

Force: [N or kg·m/s^2]

N (newton) = $\dfrac{kg\cdot m}{s^2} = \dfrac{J}{m} = \dfrac{C\cdot V}{m} = \dfrac{W\cdot s}{m}$

Temperature: [°C or K] 0°C = 273.15K

Volume: [m^3] = 1000 liters

Note: In this class, we typically use units of KJ, KPa, and KW.

SI UNITS, International System of Units

Length	meter	Temperature:	kelvin
Mass:	kilogram	Amount:	mole
Time:	second	Light intensity:	candela.
Electric current:	ampere		

ENERGY [J]

Kinetic energy $KE = \frac{1}{2}mv^2$

Potential energy $PE = mgz$

Total energy of the system $E = U + KE + PE$

U = internal energy, i.e. sensible energy (translational, rotational, vibrational), latent energy (atomic structure, melting ice), chemical energy (bonding, separating water into hydrogen & oxygen), nuclear.

THERMODYNAMIC PROPERTIES

Thermodynamic properties are related to the energy of the system, i.e. temperature, pressure, mass, volume.

Extensive properties depend on the size or extent of the system, e.g. volume, mass, total energy.

Intensive properties are independent of size, e.g. temperature, pressure, entropy, density, specific volume.

SPECIFIC PROPERTIES

Extensive properties per unit mass are called specific properties.

Specific volume $v = \frac{V}{m}$ [m^3/kg]*

Specific energy $e = \frac{E}{m}$ [kJ/kg]

Specific internal energy $u = \frac{U}{m}$ [kJ/kg]

*We have to be careful with the units for specific volume. By convention, we deal in units of kJ, kW, and kPa for many values. When specific volume or volume is included in an equation, there is often a factor of 1000 involved.

R GAS CONSTANT [kJ/(kg·K)]

$$\boxed{R = C_p - C_v}$$

C_p = specific heat at constant pressure [kJ/(kg·°C)]
C_v = specific heat at constant volume [kJ/(kg·°C)]

R **Gas Constant of Selected Materials @300K** [kJ/(kg·°C)]					
Air	0.2870	Carbon monoxide	0.2968	Methane	0.5182
Argon	0.2081	Chlorine	0.1173	Neon	0.4119
Butane	0.1433	Helium	2.0769	Nitrogen	0.2968
Carbon dioxide	0.1889	Hydrogen	4.1240	Oxygen	0.2598

PROPERTIES OF WATER

Compressed liquid: Properties for compressed liquid are insensitive to pressure. For a given temperature use the *f* subscripted values from tables A-4 and A-5, e.g. $v \approx v_f$, $u \approx u_f$, etc. However, in the case of enthalpy, $\boxed{h \approx h_f + (P - P_{sat})v}$.

Saturated phases: Properties for the saturated phases of water are determined using tables A-4 and A-5 in the back of the book and the formulas below. Note that the *fg* subscript stands for the difference between the *g* subscripted quantity and the *f* subscripted quantity, e.g. $u_g - u_f = u_{fg}$, and is provided for convenience.

Specific volume $\boxed{v = (1-x)v_f + xv_g}$ [m^3/kg]

Internal energy $\boxed{u = (1-x)u_f + xu_g = u_f + xu_{fg}}$ [kJ/kg]

Enthalpy $\boxed{h = (1-x)h_f + xh_g = h_f + xh_{fg}}$ [kJ/kg]

Entropy $\boxed{s = (1-x)s_f + xs_g = s_f + xs_{fg}}$ [kJ/(kg·K)]

Quality $\boxed{x = \frac{v - v_f}{v_g - v_f}}$ [no units]

Superheated vapor: Properties for superheated vapor are read directly from table A-6 in the back of the book.

PHASES OF WATER

The different states in which water exists are its phases. We are only concerned with the liquid and vapor states.

compressed liquid – purely liquid, at less than saturation temperature (boiling point at pressure), $v < v_f$

saturated liquid – purely liquid, but at the saturation temperature (any additional heat will cause some vaporization), $v = v_f$

saturated liquid/vapor mixture – a mixture of liquid and vapor at the temperature (and pressure) of saturation, $v_f < v < v_g$

saturated vapor – purely vapor, but at the saturation temperature (any loss of heat will cause some condensation to occur), $v = v_g$

superheated vapor – purely vapor, above the saturation temperature, $v > v_g$

ENTHALPY OF VAPORIZATION [h_{fg}]

The amount of energy needed to vaporize a unit mass of saturated liquid at a given temperature or pressure.

h_{fg} = enthalpy of vaporization [kJ/kg]

x QUALITY

The quality is a value from 0 to 1 describing the ratio of vapor mass to total mass of a pure substance. It is only applicable at saturation temperature. A quality of 0 denotes a saturated liquid and a quality of 1 denotes saturated vapor.

$$\boxed{x = \frac{m_g}{m_g + m_f}}$$

m_g = mass of the gas [kg]
m_f = mass of the fluid (liquid) [kg]

VOLUME TO MASS RELATIONSHIP

For the saturated state.

$$\boxed{V = V_f + V_g = m_f v_f + m_g v_g}$$

V = total volume [m^3]
V_f = volume of the fluid (liquid) [m^3]
V_g = volume of the gas [m^3]
m_f = mass of the fluid (liquid) [kg]
v_f = volume density of the fluid (liquid) [m^3/kg]
m_g = mass of the gas [kg]
v_g = volume density of the gas [m^3/kg]

IDEAL GAS EQUATION

The ideal gas formula assumes no intermolecular forces are involved. The ideal gas formula may be used as an approximation for the properties of gases which are a high temperatures/low pressures well out of range of their saturation (liquification) values, e.g. air at room temperature and atmospheric pressure can be considered an ideal gas.. Don't use this formula for steam, especially near saturation; use the water property tables.

$$\boxed{P\nu = RT} \text{ or } \boxed{PV = mRT}$$

In a closed system, m and R are constant, so $\dfrac{P_1V_1}{T_1} = \dfrac{P_2V_2}{T_2}$

P = pressure [kPa]
$\nu = V/m$ specific volume [m^3/kg]
V = volume [m^3]
m = mass [kg]
R = gas constant (0.287 for air) [kJ/(kg·K)]
T = absolute temperature [K] (°C + 273.15)

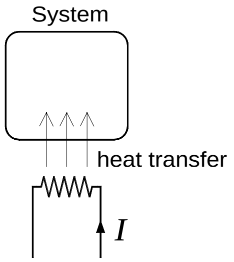
CONSTANTS

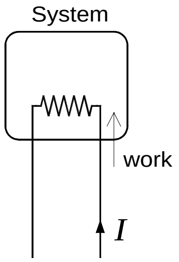
Atmospheric pressure: 101.33 kPa
Boltzmann constant: 1.380658×10^{-23} kJ/(kmol·K)
Critical point, water 22 Mpa, 374°C
Gas constant $R = R_u/M$ where M is molecular weight
(R = 287 J/(kg·K) for air)
Temperature in Kelvin: °C + 273.15

Universal gas constant: R_u = 8.314 kJ/(kmol·K)

ENERGY TRANSFER [kJ]

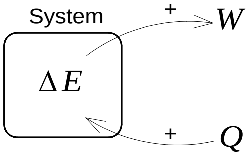
Whether energy transfer is described as heat or work can be a function of the location of the system boundary. The system boundary may be drawn to suite the problem. The area enclosed is also referred to as the **control volume**.





1st LAW OF THERMODYNAMICS [kJ]

Used in piston-cylinder problems.



Open Systems: $\boxed{Q - W = \Delta E = \Delta H + \Delta KE + \Delta PE}$

where $\Delta H = m\left(h_2 - h_1\right) = mC_{p,\,\text{avg}}\left(T_2 - T_1\right)$

Closed Systems: $\boxed{Q - W = \Delta E = \Delta U + \Delta KE + \Delta PE}$

where $\Delta U = m\left(u_2 - u_1\right) = mC_{v,\,\text{avg}}\left(T_2 - T_1\right)$

(*Closed System* means that mass does not enter or leave the system.)

$$\Delta KE = \frac{m\left(\mathrm{V}_2^{\,2} - \mathrm{V}_1^{\,2}\right)}{2000}, \qquad \Delta PE = mg\left(z_2 - z_1\right)$$

NOTE: Since the piston-cylinder is a closed system, we normally use the Closed System version of the law. An exception occurs when the piston is allowed to move as the gas expands under constant pressure. In this case, there is **boundary work** W_b, which can be included on the right-hand side of the equation by using the Open Systems version since $\Delta U + W_b = \Delta H$.

Q = net heat transfer across system boundaries, positive when flowing inward [kJ]
W = net work done in all forms, positive when flowing outward [kJ]
ΔE = net change in the total energy of the system [kJ]
ΔU = net change in the internal energy of the system [kJ]
ΔKE = net change in the kinetic energy of the system [kJ]
ΔPE = net change in the potential energy of the system [kJ]
m = mass [kg]
u = internal energy [kJ/kg]
h = enthalpy [kJ/kg]
$C_{p,\text{avg}}$ = specific heat at constant pressure, averaged for the two temperatures [kJ/(kg·°C)]
$C_{v,\text{avg}}$ = specific heat at constant volume, averaged for the two temperatures [kJ/(kg·°C)]

1st LAW, POWER VERSION [kW] for open systems

Differentiation of the 1st Law of Thermodynamics with respect to time yields the power version. Used for mixture chamber, heat exchanger, heater in a duct problems.

$$\dot{Q} - \dot{W} = \dot{m}\left(\Delta h + \Delta ke + \Delta pe\right)$$

$$\boxed{\dot{Q} - \dot{W} = \dot{m}\left[h_2 - h_1 + \frac{\mathrm{V}_2^{\,2} - \mathrm{V}_1^{\,2}}{2\left(1000\right)} + \frac{g\left(z_2 - z_1\right)}{1000}\right]}$$

where $h_2 - h_1 = C_{p,\,\text{avg}}\left(T_2 - T_1\right)$

$\dot{Q}$ = net heat transfer per unit time across system boundaries, positive when flowing inward [kW or kJ/s]
$\dot{W}$ = net work done per unit time in all forms, positive when flowing outward [kW or kJ/s]
$\dot{m}$ = mass flow rate through the control volume [kg/s] Note that to obtain this value, typically the *ideal gas equation* (p4) and the *mass flow rate* (p6) formulas will be used.
Δh = net change in enthalpy [kJ/kg]
Δke = net change in the kinetic energy per unit mass [kJ/kg]
Δpe = net change in the potential energy per unit mass [kJ/kg]
V = average flow velocity (note 1 kJ/kg = 1000 $\mathrm{m^2/s^2}$) [m/s]
g = acceleration of gravity 9.807 $\mathrm{m/s^2}$
z = elevation to some reference point [m]
$C_{p,\text{avg}}$ = specific heat at constant pressure, averaged for the two temperatures [kJ/(kg·°C)]
T_1, T_2 = temperature of the fluid at the inlet and outlet respectively [°C or K]

1^{st} LAW, UNIT-MASS VERSION [kJ/kg]

The division of the power version of the 1^{st} Law of Thermodynamics equation by the flow rate yields the unit-mass version. Used in nozzle, diffuser, turbine, and compressor problems.

Open Systems: $q - w = \Delta h + \Delta ke + \Delta pe$

$$\boxed{q - w = h_2 - h_1 + \frac{V_2^{\ 2} - V_1^{\ 2}}{2(1000)} + \frac{g(z_2 - z_1)}{1000}}$$

where $h_2 - h_1 = C_{p,\,\text{avg}}(T_2 - T_1)$

Closed Systems: $q - w = \Delta u + \Delta ke + \Delta pe$

$$\boxed{q - w = u_2 - u_1 + \frac{V_2^{\ 2} - V_1^{\ 2}}{2(1000)} + \frac{g(z_2 - z_1)}{1000}}$$

where $u_2 - u_1 = C_{v,\,\text{avg}}(T_2 - T_1)$

q = heat transfer per unit mass [kJ/kg]
w = work done per unit mass [kJ/kg]

see also BERNOULI EQUATION next.

BERNOULI EQUATION

For the steady flow of liquid through a device that involves no work interactions (such as a nozzle or pipe section), the work term is zero and we have the expression known as the Bernouli equation.

$$\boxed{0 = \upsilon(P_2 - P_1) + \frac{V_2^{\ 2} - V_1^{\ 2}}{2(1000)} + \frac{g(z_2 - z_1)}{1000}}$$

ν = V/m specific volume [m^3/kg]
P = pressure [kPa]
V = average flow velocity (note 1 kJ/kg = 1000 m^2/s^2) [m/s]
g = acceleration of gravity 9.807 m/s^2
z = elevation to some reference point [m]

H ENTHALPY [kJ]

The sum of the internal energy and the volume-pressure product. If a body is heated without changing its volume or pressure, then the change in enthalpy will equal the heat transfer. We see this more often in its per unit mass form (*see next*) called specific enthalpy but still referred to as enthalpy.

$$H = U + PV$$

U = internal energy [kJ]
P = pressure [kPa]
V = volume [m^3]

h SPECIFIC ENTHALPY [kJ/kg]

The per unit mass version of enthalpy (see previous) and often referred to as simply *enthalpy*, the exact meaning to be determined from context.

$$\Delta h = \Delta u + \nu \Delta P \qquad \text{incompressible substance}$$

$$\Delta h = \int_1^2 C_p(T)\,dT \approx C_{p,\,\text{avg}} \Delta T \qquad \text{ideal gas}$$

$$h = u + RT \qquad \text{ideal gas}$$

u = internal energy [kJ/kg]
ν = V/m specific volume [m^3/kg]
P = pressure [kPa]
$C_{p,\text{avg}}$ = specific heat at constant pressure, averaged for the two temperatures [kJ/(kg · °C)]
T = absolute temperature [K] (°C + 273.15)
R = gas constant (287 for air) [J/(kg · K)]

θ METHALPY [kJ/kg]

Methalphy means "beyond enthalpy". The factor of 1000 is used to convert m^2/s^2 to kJ/kg.

$$\theta = h + ke + pe = h + \frac{V^{\ 2}}{2 \times 1000} + gz$$

$\dot{m}$ MASS FLOW RATE [kg/s]

The rate of flow in terms of mass.

$$\boxed{\dot{m} = \frac{1}{\nu} V_1 A_1}$$

ν = V/m specific volume [m^3/kg]
V = average flow velocity (note 1 kJ/kg = 1000 m^2/s^2) [m/s]
A = cross-sectional area [m^2]

C_p, C_v SPECIFIC HEAT [kJ/(kg·°C)]

Describes the energy storage capability of a material. The energy required to raise the temperature of a unit of mass of a substance by one degree under constant pressure (C_p), or under constant volume (C_v). This can be confusing since C_p can be used in problems involving changing pressure and C_v can be used in problems involving changing volume. Note that C_p is used in calculations involving **open systems**, and C_v is used for **closed systems**. $C_p > C_v$ because at constant pressure, the system is allowed to expand when heated, requiring additional energy. The values for specific heat increase slightly with increased temperature.

$$C_p = C_v + R$$

$$u_2 - u_1 = C_v\left(T_2 - T_1\right) \qquad h_2 - h_1 = C_p\left(T_2 - T_1\right)$$

R = gas constant (287 for air) [J/(kg·K)]
u = internal energy [kJ/kg]
h = enthalpy [kJ/kg]

Example: The C_p of water at room temperature is 4.18 kJ/(kg·°C), for iron it's 0.45 kJ/(kg·°C). Therefore it takes about nine times as much energy to heat water as it does to heat iron.

$$\boxed{\text{specific heat} \times \text{mass} \times \Delta\text{temp} = \text{energy}}$$

C_p Specific Heat of Selected Materials @300K [kJ/(kg·°C)]

Air	1.005	Concrete	0.653	Iron	0.45
Aluminum	0.902	Copper	0.386	Steel	0.500
Brass	0.400	Glass	0.800	Wood, hard	1.26

k SPECIFIC HEAT RATIO [no units]

An ideal gas property that varies slightly with temperature. For monatomic gases, the value is essentially constant at 1.667; most diatomic gases, including air have a specific heat ratio of about 1.4 at room temperature.

$$\boxed{k = \frac{C_p}{C_v}}$$

C_p = specific heat at constant pressure [kJ/(kg·°C)]
C_v = specific heat at constant volume [kJ/(kg·°C)]

η_{th} THERMAL EFFICIENCY

The efficiency of a heat engine. The fraction of the heat input that is converted to net work output.

$$\boxed{\eta_{\text{th}} = \frac{W_{\text{net,out}}}{Q_{\text{in}}} = 1 - \frac{Q_{\text{out}}}{Q_{\text{in}}} = \frac{w_{\text{net}}}{q_H} = 1 - r^{1-k}}$$

$W_{\text{net,out}} = Q_H - Q_L$ = net work output [kW]
Q_{in} = heat input [kJ]
Q_{out} = heat output [kJ]

COP COEFFICIENT OF PERFORMANCE

A unitless value describing the efficiency of a refrigerator, of a heat pump.

$$\boxed{\text{COP}_{\text{Refrig.}} = \frac{\dot{Q}_L}{\dot{W}_{\text{net,in}}}} \qquad \boxed{\text{COP}_{\text{H.P.}} = \frac{\dot{Q}_H}{\dot{W}_{\text{net,in}}}}$$

$$\boxed{\text{COP}_{\text{Refrig.}} = \frac{Q_L}{Q_H - Q_L}} \qquad \boxed{\text{COP}_{\text{H.P.}} = \frac{Q_H}{Q_H - Q_L}}$$

Maximum possible COP for a refrigerator, for a heat pump:

$$\boxed{\text{COP}_{\text{Refrig.}} = \frac{1}{T_H / T_L - 1}} \qquad \boxed{\text{COP}_{\text{H.P.}} = \frac{1}{1 - T_L / T_H}}$$

$\dot{Q}_L$ = heat transfer [kW]
$\dot{W}_{\text{net,in}} = \dot{Q}_H - \dot{Q}_L$ = net work input [kW]
Q_H = heat transfer from a high temperature source [kJ]
Q_L = heat transfer from a low temperature source [kJ]
T_H = temperature of high-temperature source [K]
T_L = temperature of low-temperature source [K]

EER ENERGY EFFICIENCY RATING

An efficiency rating system used in the United States. The amount of heat removed in Btu's for 1 Wh of electricity consumed. Since 1 Wh = 3.412 Btu, this works out to:

$$\boxed{\text{EER} = 3.412\ \text{COP}_{\text{Refrig.}}}$$

Wh = watt-hour, a unit of electrical energy
Btu = British thermal unit, a unit of thermal energy
$\text{COP}_{\text{Refrig.}}$ = coefficient of performance for the refrigeration cycle, an efficiency rating [no units]

$\dot{W}$ MINIMUM POWER REQUIREMENT [kW]

The amount of power required to operate a heat pump/refrigerator.

$$\boxed{\dot{W} = \frac{\dot{Q}_L}{COP}}$$

$\dot{Q}_L$ = heat transfer [kW]
$\dot{W} = \dot{Q}_H - \dot{Q}_L$ = net work input [kW]

HEAT TRANSFER LIMIT [kW]

This expression is an equality for a reversible cycle engine (a theoretical device not realizable in practice).

$$\boxed{\frac{Q_H}{Q_L} \le \frac{T_H}{T_L}}$$

Q_H = magnitude of the heat transferred from a high-temperature source [kJ]
Q_L = magnitude of the heat transferred to a low-temperature source [kJ]
T_H = temperature of high-temperature source [K]
T_L = temperature of low-temperature source [K]

CLAUSIUS INEQUALITY

The cyclic integral of the change in heat transfer divided by the absolute temperature is always less than or equal to zero.

$$\boxed{\oint \frac{\delta Q}{T} \le 0}$$

$\oint$ = the integration is to be performed over a full cycle
δQ = the change in heat transfer [kJ]
T = absolute temperature at the inside surface of the system boundary [K]

S TOTAL ENTROPY [kJ/K]

The term *entropy* is used both for the total entropy and the entropy per unit mass s [kJ/(kg·K)]. Entropy is an *intensive* property of a system (does not depend on size).

$$\boxed{dS = \left(\frac{\delta Q}{T}\right)}$$

δQ = the change in heat transfer [kJ]
T = absolute temperature at the inside surface of the system boundary [K]

S_{gen} ENTROPY GENERATION [kJ/K]

The entropy change of a **closed system** during an irreversible process is always greater than the entropy transfer. The entropy generation is the entropy created within the system boundaries due to irreversibilities. Note that it may be necessary to extend the boundaries of a system in order to consider it a closed system.

$$\boxed{\underbrace{S_2 - S_1}_{\text{Entropy change of the system}} = \underbrace{\int_1^2 \frac{\delta Q}{T}}_{\text{Entropy transfer with heat}} + \underbrace{S_{\text{gen}}}_{\text{Entropy generation within the system}}}$$

S_1 = initial entropy [kJ/K]
S_2 = final entropy [kJ/K]
δQ = the change in heat transfer [kJ]
T = absolute temperature at the inside surface of the system boundary [K]

ENTROPY BALANCE FOR CONTROL VOLUMES

For a control volume, we must consider mass flow across the control volume boundary

$$\boxed{\underbrace{\frac{dS_{CV}}{dt}}_{\text{Rate of entropy change of CV}} = \underbrace{\sum \frac{\dot{Q}_k}{T_k}}_{\text{Rate of entropy transfer with heat}} + \underbrace{\sum \dot{m}_i s_i - \sum \dot{m}_e s_e}_{\text{Rate of entropy transport with mass}} + \underbrace{\dot{S}_{\text{gen,CV}}}_{\text{Rate of entropy generation within CV}}}$$

S_{CV} = entropy within the control volume [kJ/K]
$\dot{m}_i$ = inlet mass flow rate [kg/s]
$\dot{m}_e$ = exit mass flow rate [kg/s]
s_i = inlet entropy [kJ/(kg·K)]
s_e = exit entropy [kJ/(kg·K)]
$\dot{Q}_k$ = rate of heat transfer through the boundary at internal boundary temperature T_k [kJ/s]
T_k = absolute temperature at the inside surface of the system boundary [K]

$\dot{S}_{\text{gen}}$ RATE OF ENTROPY GENERATION [kW/K]

Most steady-state processes such as turbines, compressors, nozzles, diffusers, heat exchangers, pipes, and ducts, experience no change in their entropy. This gives the relation:

$$\boxed{\dot{S}_{\text{gen}} = \sum \dot{m}_e s_e - \sum \dot{m}_i s_i - \sum \frac{\dot{Q}_k}{T_k}}$$

For a single-stream (one inlet, one exit) steady-flow device:

$$\boxed{\dot{S}_{\text{gen}} = \dot{m}\left(s_e - s_i\right) - \sum \frac{\dot{Q}_k}{T_k}}$$

$\dot{m}_i$ = inlet mass flow rate [kg/s]

$\dot{m}_e$ = exit mass flow rate [kg/s]

s_i = inlet entropy [kJ/(kg·K)]

s_e = exit entropy [kJ/(kg·K)]

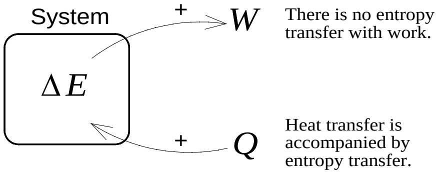
$\dot{Q}_k$ = rate of heat transfer through the boundary at temperature T_k [kJ/s]

T_k = absolute temperature at the system boundary [K]

INCREASE OF ENTROPY PRINCIPLE

The entropy of an isolated system during a process always increases or, in the limiting case of a reversible process, remains constant.

$$\boxed{\Delta S_{\text{isolated}} \geq 0}$$



There is no entropy transfer with work.

Heat transfer is accompanied by entropy transfer.

s ENTROPY PER UNIT MASS [kJ/(kg·K)]

Entropy change is caused by heat flow, mass flow, and irreversibilities. Irreversibilities always increase entropy.

$s_2 = s_1$ Isentropic process

$\boxed{s_2 - s_1 = C_{\text{avg}} \ln \frac{T_2}{T_1}}$ incompressible substances

$\boxed{\Delta s = C_{v,\text{avg}} \ln \frac{T_2}{T_1} + R \ln \frac{\nu_2}{\nu_1}}$ ideal gas

$\boxed{\Delta s = C_{p,\text{avg}} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}}$ ideal gas

$\nu = V/m$ specific volume [m^3/kg]

R = gas constant (287 for air) [J/(kg·K)]

C_{avg} = the specific heat at average temperature [kJ/(kg·K)]

C_p = specific heat at constant pressure [kJ/(kg·°C)]

C_v = specific heat at constant volume [kJ/(kg·°C)]

T_1, T_2 = initial and final temperatures [K]

P_1, P_2 = initial and final pressure [Pa]

s_{gen} ENTROPY GENERATION PER UNIT MASS [kJ/(kg·K)]

Applies to a single-stream, steady-flow device such as a turbine or compressor.

$$\boxed{s_{\text{gen}} = \left(s_e - s_i\right) - \sum \frac{q}{T}}$$

s_i = inlet entropy [kJ/(kg·K)]

s_e = exit entropy [kJ/(kg·K)]

q = heat transfer per unit mass [kJ/kg]

T = absolute temperature at the inside surface of the system boundary [K]

p THERMODYNAMIC PROBABILITY

Molecular randomness or uncertainty. The thermodynamic probability is the number of possible microscopic states for each state of macroscopic equilibrium of a system. It is related to the entropy (disorder) of the system by the **Boltzmann relation**:

$$\boxed{S = k \ln p} \quad \Rightarrow \quad p = e^{S/k}$$

S = entropy [kJ/K]

k = Boltzmann constant 1.3806×10^{-23} [kJ/(kmol·K)]

w_{rev} STEADY-FLOW WORK [kJ/kg]

One needs to know ν as a function of *P* in order to perform the integration, but when the working fluid is an incompressible fluid, the specific volume ν remains constant during the process and can be taken out of the integration. For the steady flow of a liquid through a device that involves no work (such as nozzle or a pipe section), the work term is zero

$$\boxed{w_{\text{rev}} = -\int_1^2 \nu\, dP - \Delta ke - \Delta pe}$$

$$\boxed{w_{\text{rev}} = \nu\left(P_1 - P_2\right) - \Delta ke - \Delta pe}$$

$\nu = V/m$ specific volume [m^3/kg]

HEAT ENGINES

HEAT ENGINES

The conversion of heat to work requires the use of special devices; these are called heat engines and have the following characteristics:

- They receive heat from a high-temperature source.
- They convert part of this heat to work.
- They reject the remaining waste heat to a low-temperature sink such as the atmosphere or a body of water.
- They operate on a cycle.
- They usually involve a fluid used in the transfer of heat; this is called the *working fluid*.

$W_{\text{net,out}}$ NET WORK [kJ]

The work produced by a heat engine. The net work is equal to the area bounded by the cycle as plotted on a *T-S* diagram. It is also the difference between the heat consumed by a heat engine and its waste heat, that is, the difference between heat taken from the high-temperature source and the heat deposited in the low-temperature sink.

Net work: $W_{\text{net}} = Q_H - Q_L$ [kJ]

per unit mass: $w_{\text{net}} = q_{\text{in}} - q_{\text{out}}$ [kJ/kg]

per unit time: $\dot{W}_{\text{net}} = \dot{Q}_H - \dot{Q}_L$ [kW]

Q_H = magnitude of the heat transferred from a high-temperature source [kJ]

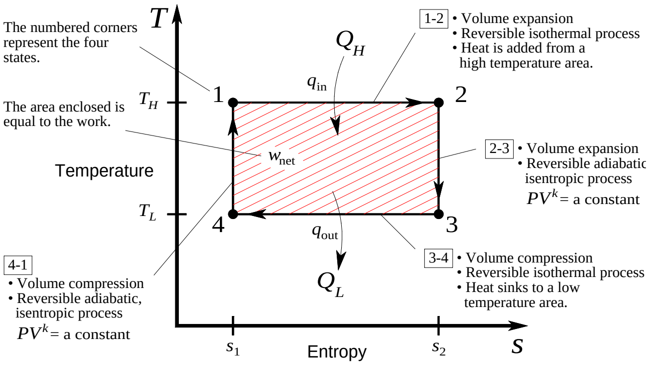
Q_L = magnitude of the heat transferred to a low-temperature source [kJ]

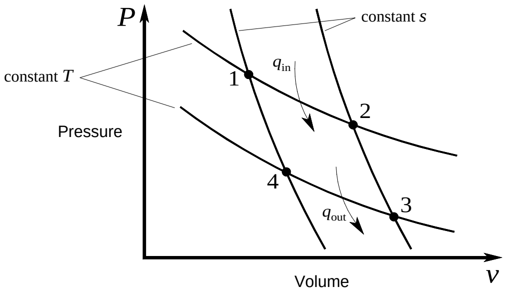
q_{in} = magnitude per kilogram of the heat transferred from a high-temperature source [kJ/kg]

q_{out} = magnitude per kilogram of the heat transferred to a low-temperature source [kJ/kg]

CARNOT CYCLE

Introduced in 1824 by French engineer Sadi Carnot, the Carnot cycle is a combination of four reversible processes that are the basis for the theoretical Carnot heat engine. The cycle forms a rectangle on the *T-s* plot. Use C_v for specific heat.





Thermal efficiency: $\eta_{\text{th,Carnot}} = 1 - \dfrac{T_L}{T_H}$

Heat transfer and work:

$$q_{\text{in}} = T_H\left(s_2 - s_1\right), \qquad q_{\text{out}} = T_L\left(s_2 - s_1\right) \text{ [kJ/kg]}$$

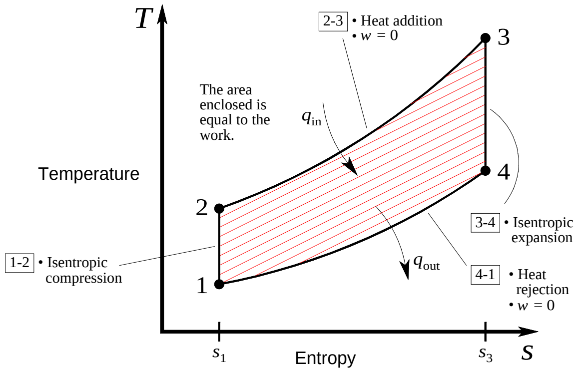
$$w_{\text{net}} = q_{\text{in}} - q_{\text{out}} \text{ [kJ/kg]}$$

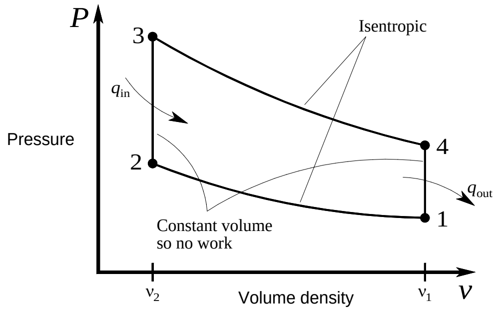
Work occurs in all 4 processes of the Carnot cycle (work is 0 for constant volume processes).

$$w_{\text{net}} = w_{12} + w_{23} + w_{34} + w_{41} \text{ [kJ/kg]}$$

IDEAL OTTO CYCLE

The Otto cycle is the model for the spark-ignition reciprocating engine. It consists of four internally reversible processes: 1) isentropic compression, 2) heat addition, 3) isentropic expansion, and 4) heat rejection.





The otto cycle is more efficient than the diesel for equal compression ratios, but the compression ratio is limited due to spontaneous ignition of the fuel at higher temperatures.

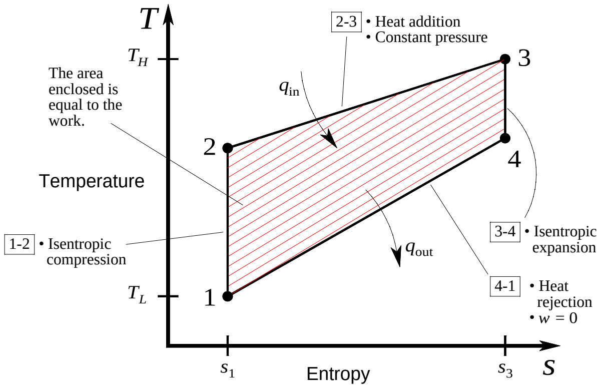
Thermal efficiency: $\eta_{\text{th,Otto}} = 1 - \frac{1}{r^{k-1}}$

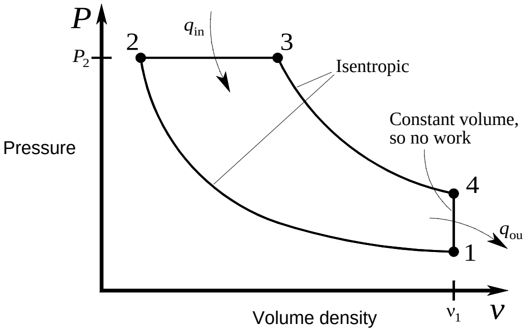
Otto cycle compression ratio is: $r = \frac{V_1}{V_2} = \frac{\nu_1}{\nu_2}$

r = compression ratio [no units]
$k = C_p/C_v$ = specific heat ratio

IDEAL DIESEL CYCLE

The Diesel cycle is the model for the compression-ignition reciprocating engine. It consists of four internally reversible processes: 1) isentropic compression, 2) heat addition, 3) isentropic expansion, and 4) heat rejection.





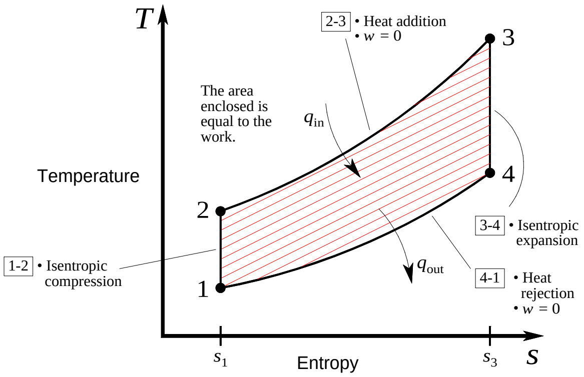
Although less efficient than the otto cycle at a given compression ratio, higher compression ratios are possible in the diesel engine, enabling greater thermal efficiency than in gasoline engines.

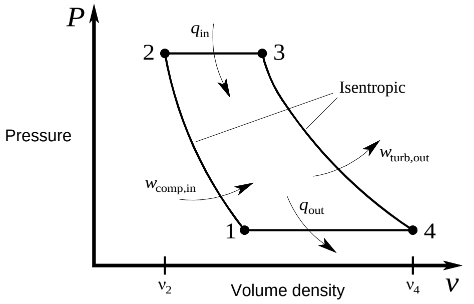
Thermal efficiency: $\eta_{\text{th,Diesel}} = 1 - \frac{1}{r^{k-1}}\left[\frac{r_c^k - 1}{k\left(r_c - 1\right)}\right]$

r = compression ratio [no units]
r_c = cutoff ratio [no units]
$k = C_p/C_v$ = specific heat ratio

IDEAL BRAYTON CYCLE

The Brayton cycle is the model used for modern **gas-turbine engines**. Although the turbine is an open system, it can be modeled by the Brayton cycle, which is a closed cycle. It consists of four internally reversible processes:1) isentropic compression, 2) heat addition under constant pressure, 3) isentropic expansion, and 4) heat rejection under constant pressure.





$$q_{\text{in}} = q_{23} = C_p\left(T_3 - T_2\right), \quad q_{\text{out}} = q_{41} = C_p\left(T_4 - T_1\right)$$

Compressor work: $w_{\text{comp,in}} = C_p\left(T_2 - T_1\right)$

Turbine work: $w_{\text{turb,out}} = C_p\left(T_3 - T_4\right)$

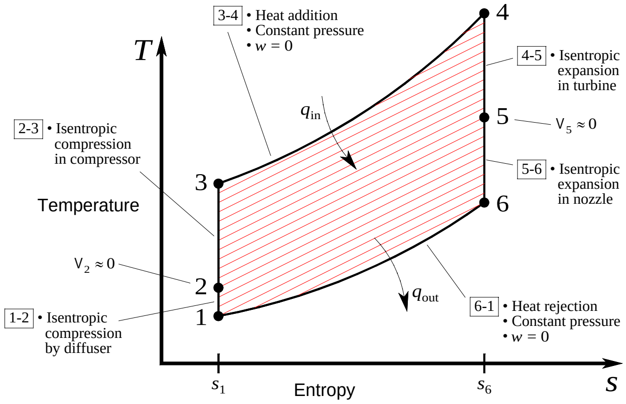
Net work: $w_{\text{net,out}} = w_{\text{turb,out}} - w_{\text{comp,in}} = q_{\text{in}} - q_{\text{out}}$

Thermal efficiency: $\eta_{\text{th,Brayton}} = 1 - \dfrac{1}{r_p^{(k-1)/k}}, \quad r_p = \dfrac{P_2}{P_1}$

C_p = specific heat at constant pressure (1.005 @ 300k) [kJ/(kg · °C)]
q = heat transfer per unit mass [kJ/kg]
w = work per unit mass [kJ/kg]
r_p = pressure ratio [no units]
$k = C_p/C_v$ = specific heat ratio

IDEAL JET-PROPULSION CYCLE

The jet-propulsion cycle is the model used for **aircraft gas-turbine engines**. It consists of six internally reversible processes:1) isentropic compression in a diffuser, 2) isentropic compression in a compressor, 3) heat addition, 4) isentropic expansion in a turbine, 5) isentropic expansion in a nozzle, and 6) heat rejection.



$$\boxed{1-2} \quad 0 = C_p\left(T_2 - T_1\right) + \frac{0 - V_1^{\,2}}{2000}$$

$$\boxed{2-3} \quad r_p = \frac{P_3}{P_2}$$

$$\boxed{4-5} \quad w_{\text{comp,in}} = w_{\text{turb,out}} \rightarrow C_p\left(T_3 - T_2\right) = C_p\left(T_4 - T_5\right)$$

$$\boxed{5-6} \quad 0 = C_p\left(T_6 - T_5\right) + \frac{V_6^{\,2} - 0}{2000}$$

Propulsive efficiency: $\eta_p = \dfrac{\dot{W}_P}{\dot{Q}_{\text{in}}} = \dfrac{w_P}{q_{\text{in}}}$

where $\dot{W}_P = \dfrac{\dot{m}\left(V_6 - V_1\right)V_1}{1000}, \quad w_P = \dfrac{\left(V_6 - V_1\right)V_1}{1000}$

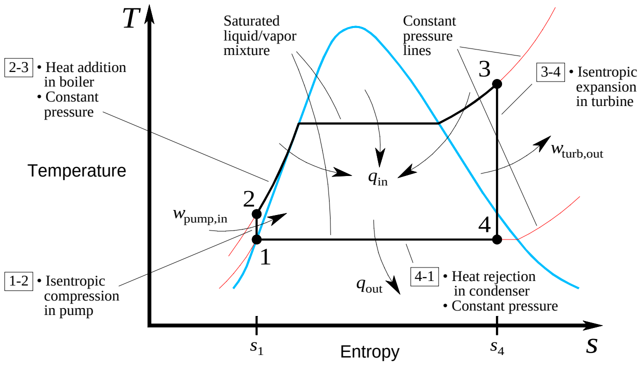
$$\dot{Q}_{\text{in}} = \dot{m} C_p\left(T_4 - T_3\right), \quad q_{\text{in}} = C_p\left(T_4 - T_3\right)$$

$$\dot{m} = \frac{1}{v} V_1 A_1$$

C_p = specific heat at constant pressure (1.005 @ 300k) [kJ/(kg · °C)]
r_p = pressure ratio [no units]
$\dot{W}_P$ = propulsive power [kW]
$\dot{Q}_{\text{in}}$ = heat transfer rate to the working fluid [kW]
$\dot{m}$ = mass flow rate [kg/s]
T = temperature [K]
P = pressure [kPa]
V = air velocity [m/s]

IDEAL RANKIN CYCLE

The Rankin cycle is the model used for vapor power plants such as **steam-turbine engines**. It consists of four internally reversible processes: 1) isentropic compression in a pump (The vertical distance between 1 and 2 is actually greatly exaggerated on the diagram below.), 2) heat addition in a boiler at constant pressure, 3) isentropic expansion in a turbine, and 4) heat rejection in a condenser at constant pressure.



$$\boxed{1-2}\quad w_{\text{pump,in}} = \nu_1\left(P_2 - P_1\right) = h_2 - h_1$$

where $h_1 = h_f\,@\,P_1,\ \ \nu_1 = \nu_f\,@\,P_1$

$$\boxed{2-3}\quad q_{\text{in}} = h_3 - h_2$$

$$\boxed{3-4}\quad w_{\text{turb,out}} = h_3 - h_4$$

Read h_3 and s_3 from Superheated Water Table based on T_3 and P_3. So $s_3 = s_4$ and $x_4 = (s_4 - s_f)/s_{fg}$. Then find $h_4 = h_f + x_4 h_{fg}$.

$$\boxed{4-1}\quad q_{\text{out}} = h_4 - h_1$$

Thermal efficiency: $\eta_p = 1 - \dfrac{q_{\text{out}}}{q_{\text{in}}} = \dfrac{w_{\text{net}}}{q_{\text{in}}}$

where $w_{\text{net}} = w_{\text{turb,out}} - w_{\text{pump,in}} = q_{\text{in}} - q_{\text{out}}$

w = work per unit mass [kJ/kg]
q = heat transfer per unit mass [kJ/kg]
h = enthalpy [kJ/kg]
$\nu = V/m$ specific volume [m^3/kg]
P = pressure [kPa]

r COMPRESSION RATIO

$$r = \frac{V_{\max}}{V_{\min}} = \frac{\nu_{\max}}{\nu_{\min}}$$

V = volume [m^3]
$\nu = V/m$ specific volume [m^3/kg]

r_c CUTOFF RATIO

The ratio of cylinder volumes after and before the combustion process. Applies to the diesel cycle.

$$r_c = \frac{V_3}{V_2} = \frac{\nu_3}{\nu_2}$$

V = volume [m^3]
$\nu = V/m$ specific volume [m^3/kg]

ISENTROPIC RELATIONS

Isentropic means that the entropy does not change. $s_2 = s_1$. The following relations apply to ideal gases:

$$\left(\frac{P_2}{P_1}\right) = \left(\frac{\nu_1}{\nu_2}\right)^k \quad \rightarrow \quad P\nu^k = \text{a constant}$$

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{\nu_1}{\nu_2}\right)^{k-1} \quad \rightarrow \quad T\nu^{k-1} = \text{a constant}$$

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} \quad \rightarrow \quad TP^{(1-k)/k} = \text{a constant}$$

$\nu = V/m$ specific volume [m^3/kg]
T_1, T_2 = initial and final temperatures [K]
P_1, P_2 = initial and final pressure [Pa]
$k = C_p/C_v$ = specific heat ratio

POLYTROPIC PROCESS

A process in which the compression and expansion of real gases have the following pressure/volume relationship.

$$\boxed{PV^n = \text{a constant}} \text{ and } \boxed{P\nu^n = \text{another constant}}$$

where n is also a constant

Work is $W = \dfrac{mR\left(T_2 - T_1\right)}{1-n}, \quad n \neq 1$

W_b BOUNDARY WORK [kJ]

The work done in a moving boundary process such as piston-cylinder expansion (positive W) and compression (negative W). The boundary work depends on the initial and final states as well as the path taken between them. In practice, it is often easier to measure work than to calculate it.

$$\boxed{W_b = \int_1^2 P\,dV}$$

For a polytropic process: $W = \dfrac{mR\left(T_2 - T_1\right)}{1-n}, \quad n \neq 1$

In order to include the boundary work in a closed system piston-cylinder operating under constant pressure, it may be necessary to use the open system equation since

$$\Delta H = \Delta U + W_b$$

V = volume [m^3]
m = mass [kg]
n = a constant
T_1, T_2 = initial and final temperatures [K]
P = pressure [kPa]
R = gas constant (0.287 kJ/(kg·K) for air) [kJ/(kg·K)]
ΔU = net change in the internal energy of the system [kJ]

AIR STANDARD ASSUMPTIONS

Since air is composed mostly of nitrogen, which undergoes few changes in the combustion chamber, internal combustion engines can be modeled as containing air only.

1) The working fluid is air that continuously circulates in a closed loop and behaves as an ideal gas.
2) All the processes are reversible.
3) The combustion process is replaced by a heat addition process from an external source
4) The exhaust process is replaced by a heat rejection process that restores the working fluid to its initial state.

MEP MEAN EFFECTIVE PRESSURE

A fictitious pressure which, if it acted on the piston during the entire power stroke, would produce the same amount of work that is produced during the actual cycle.

$$\text{MEP} = \frac{W_{\text{net}}}{V_{\max} - V_{\min}} = \frac{w_{\text{net}}}{\nu_{\max} - \nu_{\min}}, \text{ where } \nu = \frac{RT}{P}$$

HEAT TRANSFER

HEAT TRANSFER

Energy transport due to temperature difference.

- **Conduction** – diffusion in a material. In liquids and gases, conduction is due to molecular collisions. In solids, it is due to 1) molecular vibration in the lattice and 2) energy transport by free electrons.
- **Convection** – by bulk motion of the fluid
- **Thermal radiation** – by electromagnetic waves; doesn't require a medium

The three mechanisms for heat transfer cannot all operate simultaneously in a medium.

Gases - are usually transparent to radiation, which can occur along with either conduction or convection but not both. In gases, radiation is usually significant compared to conduction, but negligible compared to convection.

Solids - In opaque solids, heat transfer is only by conduction. In semitransparent solids, heat transfer is by conduction and radiation.

Fluids - In a still fluid, heat transfer is by conduction; in a flowing fluid, heat transfer is by convection. Radiation may also occur in fluids, usually with a strong absorption factor.

Vacuum - In a vacuum, heat transfer is by radiation only.

HEAT TRANSFER AND WORK

Heat transfer and work are *interactions* between a system and its surroundings. Both are recognized as they cross the boundaries of a system. Heat and work are **transfer phenomena**, not properties. They are associated with a process, not a state. Both are **path functions**, meaning that their magnitudes depend on the path taken as well as the end states.

k THERMAL CONDUCTIVITY [W/(m·°C)]

A measure of the ability of a material to conduct heat. k varies with temperature, but we will consider it constant in this class. The conductivity of most solids falls with increasing temperature but for most gases it rises with increasing temperature. For example, the conductivity of copper at 200K is 413, and at 800K is 366. The conductivity of air at 200K is 0.0181, and at 800K is 0.0569. The change in conductivity becomes more dramatic below 200K.

Thermal Conductivity of Selected Materials @300K [W/(m·°C)]					
Air	0.0261	Copper	401	Human skin	0.37
Aluminum	237	Diamond	2300	Iron	80.2
Brick	0.72	Fiberglass insul.	0.04	Mercury	8.9
Carbon dioxide	0.0166	Glass	1.4	Plywood	0.12
Concrete	1.4	Gypsum	0.17	Water	0.608
Concrete block	1.1	Helium	0.150	Wood (oak)	0.17

ρC_p HEAT CAPACITY [J/(m^3 · °C)]

The heat storage capability of a material on a per unit volume basis. The term C_p alone expresses heat capacity per unit mass [J/(kg · °C)]. ρ is density [kg/m^3].

α THERMAL DIFFUSIVITY [m^2/s]

The ratio of heat conducted to the heat stored per unit volume.

$$\partial = \frac{\text{heat conducted}}{\text{heat stored}} = \frac{k}{\rho C_p}$$

k = thermal conductivity [W/(m · °C)]
ρC_p = heat capacity [J/(m^3 · °C)]

R-value THERMAL RESISTANCE

The *R*-value is the thermal resistance of a material per unit area. In the United States, the *R*-value is commonly expressed without units, e.g. R-19 and R-30. These values are obtained by dividing the thickness of the material in feet by its thermal conductivity in units of Btu/(h · ft · °F). This results in units of (h · ft^2 · °F)/Btu.

$$\boxed{R_{\text{value}} = \frac{L}{k}} \quad [(\text{h} \cdot \text{ft}^2 \cdot {}^\circ\text{F})/\text{Btu}]$$

L = thickness of the material [feet]
k = thermal conductivity [Btu/(h · ft · °F)]

R_t THERMAL RESISTANCE [°C/W]

The resistance of a surface (convection resistance) or of a material (conduction resistance) to heat transfer.

Conduction resistance: $\boxed{R_{t\,\text{cond.}} = \dfrac{L}{kA}}$ [°C/W]

Convection resistance: $\boxed{R_{t\,\text{conv.}} = \dfrac{1}{hA}}$ [°C/W]

L = thickness of the material [m]
k = thermal conductivity [W/(m · °C)]
h = convection heat transfer coefficient [W/(m^2 · °C)]
A = area [m^2]

R_t THERMAL RESISTANCE OF A CYLINDRICAL SHAPE [°C/W]

The thermal resistance per unit length of a cylindrical shape, e.g. pipe or pipe insulation.

Through the material: $\boxed{R_{t\,\text{cond.}} = \dfrac{\ln \frac{r_2}{r_1}}{2\pi l k}}$ [°C/W]

Across a boundary: $\boxed{R_t = \dfrac{1}{(2\pi r l)h}}$ [°C/W]

l = length [m]
k = thermal conductivity [W/(m · °C)]
h = convection heat transfer coefficient [W/(m^2 · °C)]
r_1 = inner radius [m]
r_2 = outer radius [m]

R_c CONTACT RESISTANCE [°C/W]

When two surfaces are pushed together, the junction is typically imperfect due to surface irregularities. Numerous tiny air pockets exist at the junction resulting in an abrupt temperature change across the boundary. The value is determined experimentally and there is considerable scatter of data.

h_{conv} CONVECTION HEAT TRANSFER COEFFICIENT [W/(m^2 · °C)]

The convection heat transfer coefficient is not a property of the fluid. It is an experimentally determined parameter whose value depends on many variables such as surface geometry, the nature of the fluid motion, the properties of the fluid, and the bulk fluid velocity.

Typical Values of the Convection Heat Transfer Coefficient	
Free convection of gases	2-25
Free convection of liquids	10-1000
Forced convection of gases	25-250
Forced convection of liquids	50-20,000
Boiling and condensation	2500-100,000

h_{rad} RADIATION HEAT TRANSFER COEFFICIENT [W/(m^2 · °C)]

The radiation heat transfer coefficient depends strongly on temperature while the convection heat transfer coefficient does not. A surface exposed to the surrounding air involves parallel convection and radiation, so that the two are combined by adding. It is possible for them to be in opposite directions, involving a subtraction.

$$h_{\text{rad}} = \varepsilon\sigma\left(T_s^2 + T_{\text{surr}}^2\right)\left(T_s + T_{\text{surr}}\right)$$

$\dot{Q}$ HEAT TRANSFER RATE [W]

Conduction: $\boxed{\dot{Q}_{\text{cond}} = -kA\frac{dT}{dx} = -kA\frac{T_2 - T_1}{L}}$*

Convection: $\boxed{\dot{Q}_{\text{conv}} = hA\left(T_2 - T_1\right)}$

Radiation: $\boxed{\dot{Q}_{\text{rad}} = \sigma T^4}$

***Fourier's law of heat conduction**, introduced in 1822 by J. Fourier.

k = thermal conductivity [W/(m · °C)]

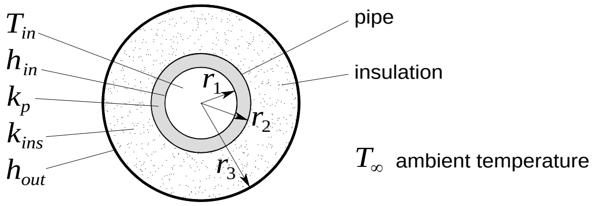
h = convection heat transfer coefficient [W/(m^2 · °C)]

A = surface area normal to the direction of heat transfer [m^2]

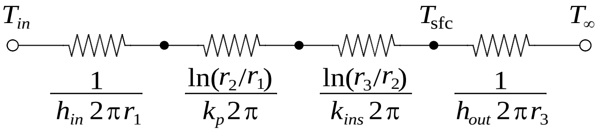
T_1, T_2 = temperature [°C or K]

L = thickness of the material [m]

$\dot{Q}$ HEAT TRANSFER THROUGH AN INSULATED PIPE [W]



Resistance model:



$$\boxed{\dot{Q} = \frac{T_{in} - T_\infty}{\frac{1}{h_{in}2\pi r_1} + \frac{\ln\left(r_2/r_1\right)}{k_p 2\pi} + \frac{\ln\left(r_3/r_2\right)}{k_{ins} 2\pi} + \frac{1}{h_{out}2\pi r_3}}}$$

h_{in} = convection heat transfer coefficient, inner wall [W/(m^2 · °C)]

k_p = thermal conductivity of the pipe material [W/(m · °C)]

k_{ins} = thermal conductivity of the insulation [W/(m · °C)]

h_{out} = convection heat transfer coefficient of the insulation surface [W/(m^2 · °C)]

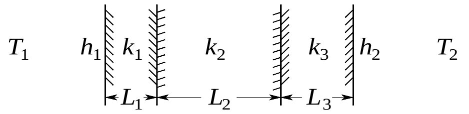
$\dot{q}$ HEAT FLUX [W/m²]

Heat transfer per unit time per unit area.

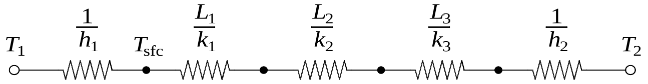
Through a material: $\boxed{\dot{q}=\dfrac{T_1-T_2}{L/K}}$

Across a boundary: $\boxed{\dot{q}=\dfrac{T_1-T_2}{1/h}}$

Example: For a composite wall with convective effects h_1 and h_2 (wind) at the surfaces:



The problem can be modeled as a series of resistances:



Note that for the **thermal resistances** $\frac{1}{h}$ and $\frac{L}{k}$ we are using units of [(m² · °C)/W], not [°C/W].

The heat flux is: $\dot{q}=\dfrac{T_1-T_2}{\dfrac{1}{h_1}+\dfrac{L_1}{k_1}+\dfrac{L_2}{k_2}+\dfrac{L_3}{k_3}+\dfrac{1}{h_2}}$

Now the surface temp can be found from the relation:

$$\dot{q}=\frac{T_1-T_{\text{sfc}}}{1/h_1}$$

k = thermal conductivity [W/(m · °C)]
h = convection heat transfer coefficient [W/(m² · °C)]
A = surface area [m²]
T_{sfc} = surface temperature [°C or K]
T_1, T_2 = temperature [°C or K]
L = thickness of the material [m]

r_{cr} CRITICAL RADIUS [m]

Adding insulation to a cylindrical object can reduce heat loss due to the insulating effect, but there is also an increase in surface area. In the case of a small diameter cylinder, the heat loss due to increased surface area may outweigh the benefit of the added insulation. At some point, the two effects are equal and this is called the critical radius of insulation for a cylindrical body. This is the outer radius that includes the thickness of the insulation.

$$\boxed{r_{\text{cr,cylinder}}=\frac{k}{h}}$$

k = thermal conductivity of the insulation [W/(m · °C)]
h = external convection heat transfer coefficient [W/(m² · °C)]

$\dot{g}$ HEAT GENERATION IN A RESISTIVE WIRE [W/m³]

Per unit volume: $\boxed{\dot{g}=\dfrac{I^2R}{\pi r_0^2L}}$ [W/m³]

Heat generation = heat dissipation:

$$\boxed{\dot{Q}=I^2R=\dot{g}V=\dot{g}\left(\pi r_0^2L\right)=h\left(T_s-T_\infty\right)\left(2\pi r_0L\right)}$$

Surface temperature: $\boxed{T_s=T_\infty+\dfrac{\dot{g}r_0}{2h}}$ [K]

Temperature at the center: $\boxed{T_0=T_s+\dfrac{\dot{g}r_0^2}{4k}}$ [K]

I = current [A]
R = resistance [Ω]
r_0 = outer radius of wire [m]
L = length of wire [m]
V = volume [m³]
h = convection heat transfer coefficient [W/(m² · °C)]
T_∞ = ambient temperature [K]

L_c CHARACTERISTIC LENGTH [m]

The equivalent penetration distance of a shape in its thermodynamic application. For example a large flat plate immersed in a fluid would have a characteristic length equal to *half of its thickness*, unless the temperature difference existed across the plate in which case the characteristic length would be equal to the *total thickness*. The characteristic length is used in finding the Biot number.

$\boxed{L_c=\dfrac{V}{A}}$

Cylinder: $L_c=\frac{1}{2}r$

Sphere: $L_c=\frac{1}{3}r$

V = volume [m³]
A = surface area [m²]

Bi BIOT NUMBER

The ratio of the internal resistance of a body to heat conduction to its external resistance to heat convection. If the Biot number is very small, it means that the internal temperature is uniform so that it is possible to use *lumped system analysis* in determining thermal behavior.

$$\boxed{\text{Bi}=\frac{hL_c}{k}}$$

V = volume [m³]
h = external convection heat transfer coefficient [W/(m²·°C)]
L_c = characteristic length of the body [m]
k = thermal conductivity of the insulation [W/(m·°C)]

b THERMAL TIME CONSTANT [s^{-1}]

The thermal time constant is used in lumped system analysis (*see next*) to describe the (exponential) rate at which a body approaches thermal equilibrium. It is the inverse time it takes for the temperature-equilibrium gap to reduce to $1/e$ (approximately $1/3$) it's previous amount. For example if a 400°C object is placed in a 100°C medium, it will cool to 200°C in approximately $1/b$ seconds.

$$\boxed{b=\frac{h}{\rho C_p L_c}}$$

h = external convection heat transfer coefficient [W/(m^2·°C)]
ρ = volume density [kg/m^3]
C_p = specific heat at constant pressure (1.005 @ 300k) [kJ/(kg · °C)]
L_c = characteristic length of the body [m]

LUMPED SYSTEM ANALYSIS

Lumped system analysis assumes the interior temperature of a body to be uniform throughout the body. The method may be used when the Biot number is small.

$$\mathrm{Bi} \le 0.1$$

When this condition is met, the temperature variation across the internal area of the body will be slight. The following relation may be used:

$$\boxed{\frac{T(t)-T_\infty}{T_i-T_\infty}=e^{-bt}}. \text{ where } b=\frac{h}{\rho C_p L_c}$$

To find the time at which the temperature change from T_i to T_∞ is 99% complete, set $e^{-bt} = 0.01$.

$T(t)$ = temperature at time t [°C]
T_∞ = temperature of the surroundings [°C]
T_i = initial temperature of the body [°C]
b = time constant (*see previous*) [s^{-1}]
t = time [s]
h = external convection heat transfer coefficient [W/(m^2·°C)]
ρ = volume density [kg/m^3]
C_p = specific heat at constant pressure (1.005 @ 300k) [kJ/(kg · °C)]
L_c = characteristic length of the body [m]

STEPHAN-BOLTZMANN LAW

The idealized surface that emits radiation at the maximum rate is called a **blackbody**. The maximum radiation that can be admitted from a body at temperature T_s is given by the Stephan-Boltzmann law.

$$\dot{q}=\sigma T_s^4 \quad [\mathrm{W/m^2}]$$

$$\dot{Q}_{\max}=\sigma A T_s^4 \quad [\mathrm{W}]$$

σ = Boltzmann constant, 5.6705×10^{-8} W/(m^2 · K^4)
A = surface area [m^2]
T_s = surface temperature [K]

ε EMISSIVITY [no units]

Radiation from real surfaces is somewhat less than radiation from a blackbody (idealized heat-radiating surface). The emissivity of a surface is the factor that relates its ability to radiate heat to that of an ideal surface.

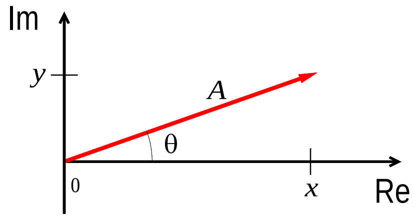
$$\dot{q}=\varepsilon\sigma T_s^4 \quad [\mathrm{W/m^2}]$$

$$\dot{Q}_{\max}=\varepsilon\sigma A T_s^4 \quad [\mathrm{W}]$$

ε = emissivity $0\le\varepsilon\le1$ [no units]
σ = Boltzmann constant, 5.6705×10^{-8} W/(m^2 · K^4)
A = surface area [m^2]
T_s = surface temperature [K]

GENERAL MATHEMATICAL

$x + \mathrm{j}\,y$ COMPLEX NUMBERS



$$x+\mathrm{j}y = Ae^{\mathrm{j}\theta} = A\cos\theta + \mathrm{j}A\sin\theta$$

$$\mathrm{Re}\{x+\mathrm{j}y\} = x = A\cos\theta$$

$$\mathrm{Im}\{x+\mathrm{j}y\} = y = A\sin\theta$$

$$\text{Magnitude}\{x+\mathrm{j}y\} = A = \sqrt{x^2+y^2}$$

$$\text{Phase}\{x+\mathrm{j}y\} = \theta = \tan^{-1}\frac{y}{x}$$

$$j = e^{j\frac{\pi}{2}}$$

The magnitude of a complex number may be written as the absolute value.

$$\text{Magnitude}\{x+\mathrm{j}y\} = |x+\mathrm{j}y|$$

The square of the magnitude of a complex number is the product of the complex number and its **complex conjugate**.

$$|x+\mathrm{j}y|^2 = (x+\mathrm{j}y)(x+\mathrm{j}y)^* = (x+\mathrm{j}y)(x-\mathrm{j}y)$$

SERIES

$$\sqrt{1+x} \approx 1+\frac{1}{2}x\,,\quad |x| \ll 1$$

$$\frac{1}{\sqrt{1+x}} \approx 1-\frac{x}{2}+\frac{3x^2}{8}-\frac{5x^3}{16}+\frac{35x^4}{128}-\cdots,\quad -\tfrac{1}{2}<x<\tfrac{1}{2}$$

$$\frac{1}{1-x^2} \approx 1+x^2+x^4+x^6+\cdots,\quad -\tfrac{1}{2}<x<\tfrac{1}{2}$$

$$\frac{1}{(1-x)^2} \approx 1+2x+3x^2+4x^3+\cdots,\quad -\tfrac{1}{2}<x<\tfrac{1}{2}$$

$$\frac{1}{1+x} \approx 1-x+x^2-x^3+\cdots,\quad -\tfrac{1}{2}<x<\tfrac{1}{2}$$

$$\frac{1}{1-x} \approx 1+x+x^2+x^3+\cdots,\quad -\tfrac{1}{2}<x<\tfrac{1}{2}$$

EULER'S EQUATION

$$\boxed{e^{j\phi} = \cos\phi + j\sin\phi}$$

and $j = e^{j\frac{\pi}{2}}$

HYPERBOLIC FUNCTIONS

$$j\sin\theta = \sinh(j\theta)$$

$$j\cos\theta = \cosh(j\theta)$$

$$j\tan\theta = \tanh(j\theta)$$

PHASOR NOTATION

To express a derivative in phasor notation, replace $\frac{\partial}{\partial t}$ with $j\omega$. For example, the Telegrapher's equation $\frac{\partial V}{\partial z} = -L\frac{\partial I}{\partial t}$

becomes $\frac{\partial V}{\partial z} = -Lj\omega I$.

SPHERE

$$\text{Area} = \pi d^2 = 4\pi r^2 \qquad \text{Volume} = \tfrac{1}{6}\pi d^3 = \tfrac{4}{3}\pi r^3$$

GRAPHING TERMINOLOGY

With *x* being the horizontal axis and *y* the vertical, we have a graph of ***y*** versus ***x*** or ***y* as a function of *x***. The *x*-axis represents the **independent variable** and the *y*-axis represents the **dependent variable**, so that when a graph is used to illustrate data, the data of regular interval (often this is time) is plotted on the *x*-axis and the corresponding data is dependent on those values and is plotted on the *y*-axis.

GLOSSARY

adiabatic Describes a process in which there is no heat transfer. This could either be a well-insulated system or a system that is at the same temperature as the surroundings. Adiabatic is not the same as isothermal because in an adiabatic process the temperature *can* change by other means such as work.

chemical energy The internal energy associated with the atomic bonds in a molecule.

critical point The pressure and temperature at which a substance will change state (between liquid and vapor) instantly without an intermediate state of saturated liquid/vapor mixture. The critical point for water is 22 MPa, 374°C.

enthalpy (H) The sum of the internal energy and the volume-pressure product. If a body is heated without changing its volume or pressure, then the change in enthalpy will equal the heat transfer. Units of kJ. Enthalpy also refers to the more commonly used specific enthalpy or enthalpy per unit mass h, which has units of $\mathrm{kJ/kg}$.

entropy (s) The unavailable energy in a thermodynamic system, also considered a measure of the disorder of a system. Increasing heat energy increases entropy; increasing pressure yields an increase in energy but little or no increase in entropy. The entropy of a material will be highest in the gas phase and lowest in the solid phase. Units of $\mathrm{kJ/(kg \cdot K)}$.

heat engine A heat engine operates on a cycle, receives heat from a high-temperature source, converts part of it to work, and rejects the waste heat to a low-temperature sink.

internal energy (u) The sum of all microscopic forms of energy of a system.

irreversible process A process that is not reversible—duh. There will be entropy generation in an irreversible process. Factors that can cause a process to be irreversible include friction, unrestrained expansion, mixing of two gases, heat transfer across a finite temperature difference, electric resistance, inelastic deformation of solids, and chemical reactions.

isentropic Having constant entropy. An isentropic process is an internally reversible adiabatic process in which the entropy does not change.

isothermal Describes a process in which temperature remains constant.

latent energy The internal energy associated with the phase of a system. When sufficient energy is added to a liquid or solid the intermolecular forces are overcome, the molecules break away, forming a gas. Because of this, the gas is at a higher energy level than the solid or liquid. The internal energy associated with phase change is the latent energy.

nuclear energy The rather large amount of internal energy of a system associated with the bonds within the nucleus of the atom itself.

pascal A unit of pressure in the MKS system equal to one Newton per square meter.

path function A process result whose magnitude depends on the path taken. Heat transfer and work are examples of path functions. Path functions have inexact differentials designated by the symbol δ.

point function A property whose magnitude depends only upon the state and not on the path taken. Point functions have exact differentials designated by the symbol d.

pure substance Composed of a single homogeneous chemical species, e.g. water (even with ice). Air is often considered a pure substance in its gaseous form. However, air in the form of a liquid/gas mixture is not a pure substance since the liquid and gas do not have the same chemical composition. This is because the various gases that comprise air liquefy at different temperatures.

reversible process An idealized process that can be reversed without leaving any trace on the surroundings. That is, the net heat and work exchanged between the system and the surroundings is zero for the combined process and reverse process. There is zero entropy generation in a reversible process.

sensible energy The portion of the internal energy of a system associated with the kinetic energies of the molecule. These kinetic energies are 1) translational kinetic energy, 2) rotational kinetic energy, and 3) vibrational kinetic energy. The degree of activity of the molecules is proportional to the heat of the gas. Higher temperatures mean greater kinetic energy and therefore higher internal energy.